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Inter-numerology interference in OFDM-IM systems

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Abstract

In 5G and beyond communication systems, distinct numerologies can coexist to serve diverse requirements for users and applications. However, the inter-numerology interference (INI) is a main challenge that significantly impacts the system performance. Therefore, the performance under INI has become an essential evaluation metric for the suitability of the different transmission schemes in the future communication systems. This paper analyzes the impact of INI on the performance of orthogonal frequency division multiplexing with index modulation (OFDM-IM) systems. Specifically, an analytical expression of the INI level in OFDM-IM systems is presented as a function of the subcarrier activation ratio (SAR) and subcarrier activation probability (SAP). Furthermore, aiming at reducing the INI level, an adaptive subcarrier mapping scheme (SMS) is proposed based on the conventional combinatorial mapping scheme. Moreover, analysis and evaluation of SAR and SAP are performed regarding the requirements of 5G and beyond services. It is proved that the INI level in OFDM-IM systems is highly dependent not only on the number of active subcarriers but also on their position in an OFDM block.

1 | INTRODUCTION

Next-generation communication systems aim to meet the diverse demands of emerging applications and use cases, such as massive connectivity, ultra-reliability and low latency, in contrast to fourth generation (4G) systems where a high data rate was the primary key performance indicator [1, 2]. The conventional orthogonal frequency division multiplexing (OFDM) with a fixed subcarrier spacing (SCS) hampers the support of diverse requirements in fifth generation (5G) New Radio (NR) and beyond networks. Therefore, *multi-numerology* concept is adopted by Third Generation Partnership Project (3GPP) standardization [3, 4]. However, the coexistence of multiple numerologies within a frame leads to inter-numerology interference (INI) due to loss of orthogonality between OFDM signals.

In the literature, extensive research is ongoing for the analysis and mitigation of INI impact under various scenarios. In [5], authors have developed a theoretical model for the analysis of INI in windowed OFDM systems. The INI effect on a particular subcarrier has been classified based on its power as dominant or non-dominant. The dominant INI is removed by successive interference cancellation (SIC), while the remaining interference is exploited for soft decoding of coded bits at the receiver. Kihero et al. have built a general mathematical framework to assess the INI effect in conventional OFDM considering individual and common cyclic prefix (CP) utilization, where common CP corresponds to the addition of a single CP for multiple OFDM symbols. The use of common CP results in INI-free subcarriers on the user with narrow SCS (UN) while its remaining subcarriers face higher INI than that of individual

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CP case [6]. In line with this, in [7], Memisoglu et al. have introduced a new way of guard band implementation between adjacent numerologies for avoiding INI. With the aid of common CP, INI-free subcarriers located in the guard band are also utilized for data transmission in order to minimize spectral efficiency loss. In [8], authors have developed a theoretical model for INI analysis in multiple-input multiple-output (MIMO)-OFDM systems and proposed a cancellation technique based on the channel knowledge. Choi et al. have explored the INI effect under non-orthogonal multiple accessing (NOMA) [9]. The increase in interference variance is observed when different numerologies are assigned to the overlapping users compared to conventional NOMA with a single numerology. Moreover, in [10], power allocation mechanism for adjacent users has been proposed in order to suppress INI impact in OFDM systems.

In recent years, OFDM with index modulation (OFDM-IM) have become more appealing due to its flexible implementation for serving different user requirements [11, 12]. In the literature, it is shown that OFDM-IM outperforms the classical OFDM under non-orthogonal transmission, doubly dispersive channels, asynchronous transmission and hardware impairments including peak-to-average-power ratio, carrier frequency offset, and in-phase and quadrature imbalance [13–20]. However, INI analysis for OFDM-IM is missing in the literature. Although the performance of filtered-OFDM with IM is presented in [21] via simulation results, flexible structure of OFDM-IM is not exploited for the INI control in OFDM systems. In this sense, the main contributions of this paper are listed as follows

- Here, the impact of INI on OFDM-IM system is analyzed and mathematically formulated through the two unique features of the OFDM-IM, namely, subcarrier activation ratio (SAR) and subcarrier activation probability (SAP).
- We prove that the use of IM in systems with multiple numerologies can cause poorer system performance than traditional OFDM systems if the flexible structure of OFDM-IM is not exploited.
- In order to minimize the INI effect in OFDM-IM systems through adaptive use of the IM concept, we have adapted the subcarrier mapping schemes (SMSs) for adjacent users to optimize both SAR and SAP for minimizing the INI effects. The proposed method allows the UN to use the conventional SMS scheme, called combinatorial (COMB) method, while another modified SMS is proposed to be used by the user with wide SCS (UW).
- Both theoretical analysis and simulation results show that adaptive use of the unique features of OFDM-IM significantly improves system performance in the presence of INI and thus outperforms conventional OFDM transmission.

The rest of the paper is organized as follows. Section 2 presents the multi-numerology system with OFDM-IM. Section 3 introduces the proposed SMS for combatting INI in OFDM systems. Section 4 provides theoretical INI analysis for OFDM-IM systems considering SAR and SAP. Section 5 presents numerical and simulation results and our discussions. Lastly, Section 6 contains the concluding remarks.[1]

2 | MULTI-NUMEROLOGY SYSTEM WITH OFDM-IM

OFDM-IM performs partial subcarrier activation for the transmission of data bits not only by the utilized subcarriers but also by their position in a block [15]. Incoming v bits are divided into $G = v/p$ subblocks, where p corresponds to the number of bits per subblock. Later, p bits are divided into two groups as p_1 and p_2 bits for the transmission of index bits and data symbols, respectively. For total N_{tot} subcarriers, each subblock consists of $b = N_{tot}/G$ subcarriers, where a out of b subcarriers are activated per subblock. Therefore, the number of transmitted bits by the activated subcarriers and their indices correspond to $p_2 = a\log_2(Q)$ and $p_1 = \lfloor \log_2 C(b,a) \rfloor$ bits, respectively, where Q is modulation order. Note that each possible p_1 information bits are mapped to unique a subcarriers by SMSs available in the literature [15, 22]. As a result, $Gp = G(\lfloor \log_2 C(b,a) \rfloor + a\log_2(Q))$ bits are transmitted by an OFDM-IM block. The active subcarriers' indices for gth OFDM-IM subblock are shown as

$$\Gamma^g = [i^g(1), \dots, i^g(\gamma), \dots, i^g(a)]_{1\times a}, \tag{1}$$

where $i^g(\gamma) \in [1, 2, \dots, b]$. Thus, data symbols in frequency domain for gth OFDM-IM subblock equals

$$X_{IM}^g(k) = \begin{cases} 0, & i^g(k) \notin \Gamma^g, \\ s, & i^g(k) \in \Gamma^g, \end{cases} \tag{2}$$

where $s \in \mathcal{S}$ denoting the set of Q-ary symbols $\mathcal{S} = \{s_0, \dots, s_{Q-1}\}$. Later, G OFDM-IM subblocks are concetaned to generate the frequency domain signal as $\mathbf{X}_{IM} = [\mathbf{X}_{IM}^1, \mathbf{X}_{IM}^2, \dots, \mathbf{X}_{IM}^G]$. The remaining procedure is the same as OFDM. After inverse fast fourier transform (IFFT) and CP addition, the signal is transmitter over a wireless channel. Moreover, differently from OFDM, OFDM-IM with frequency domain interleaving, that is, OFDM with interleaved subcarrier IM (OFDM-ISIM) significantly improves the bit error rate (BER) performance through vanishing the correlation between the active subcarriers [23].

In multi-numerology OFDM systems, the available spectrum $B = N\Delta f_{UN} = M\Delta f_{UW}$ Hz is shared between the existing U users, where Δf_{UN} and Δf_{UW} refer to SCSs of UN and UW, respectively. Thus, the ratio between the numerologies is $R = \Delta f_{UW}/\Delta f_{UN}$. Each user utilizes the spectrum with the ratio of r_u, given that $u = \{1, 2, \dots, U\}$. In this study, it is assumed that $U = 2$, and the spectrum is equally utilized by UN and UW, that is, $r_1 = r_2 = 1/2$. Please note that zero guard band exists between the adjacent numerologies. Considering

[1] Capital and lowercase boldface letters are used for frequency and time domain signals, respectively. $C(b,a)$ denotes the binomial coefficient and $\lfloor . \rfloor$ is the floor function.

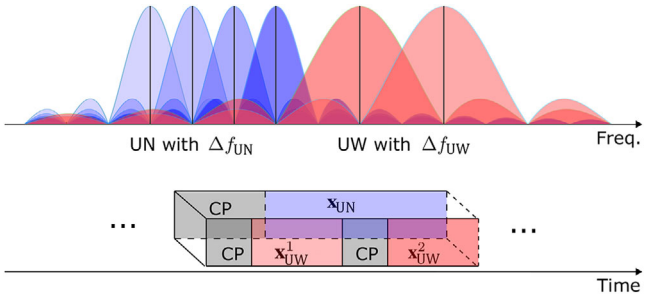


FIGURE 1 The coexistence of two numerologies in OFDM systems, while $R = \Delta f_{UW}/\Delta f_{UN} = 2$

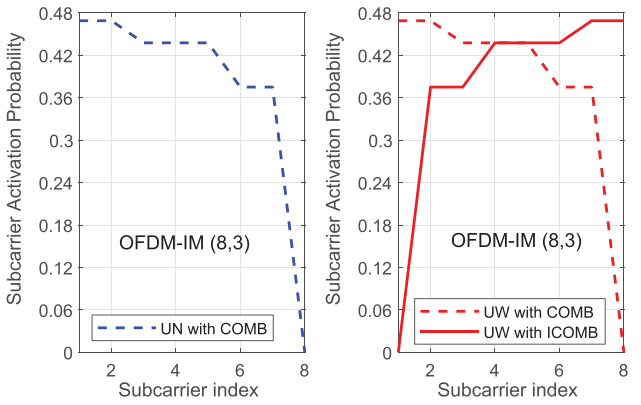


FIGURE 2 SAP for OFDM-IM with (8,3) in the case of COMB and ICOMB SMS utilization

two users, time and frequency visualization of OFDM systems with multi-numerology are illustrated in Figure 1. Let us assume that $\mathbf{X}_{UN}$ and $\mathbf{X}_{UW}$ represent the frequency domain data symbols for UN with OFDM-IM and UW with OFDM-IM, respectively. In this regard, after IFFT process, time domain signal for UN equals

$$x_{UN}(n) = \frac{N}{\sqrt{NA_{UN}}} \sum_{k=0}^{N-1} X_{UN}(k) e^{\frac{j2\pi nk}{N}}, \; 1 \le n \le N, \tag{3}$$

where

$$X_{UN}(k) = \begin{cases} \text{if } 0 \le k \le Nr_1 - 1, & \begin{cases} 0, & i^g(\lambda) \notin \Gamma^g_{UN} \\ s, & i^g(\lambda) \in \Gamma^g_{UN} \end{cases} \\ \text{if } Nr_1 \le k \le Nr_1 - 1, 0 \end{cases} \tag{4}$$

and $A_{UN} = \frac{a_{UN}}{b_{UN}}$ is the SAR for UN. a_{UN} is the number of active subcarrier within a subblock, while b_{UN} correspond to the subblock size. Due to wide SCS, FFT/IFFT size of UW corresponds to $M = N/R$. Therefore, a single OFDM symbol for UW is expressed as

$$x^{\alpha}_{UW}(m) = \frac{M}{\sqrt{MA_{UW}}} \sum_{l=0}^{M-1} X_{UW}(l) e^{\frac{j2\pi ml}{M}}, 1 \le m \le M, \tag{5}$$

where

$$X_{UW}(l) = \begin{cases} \text{if } 0 \le l \le Mr_2 - 1, \; 0 \\ \text{if } Mr_2 \le l \le M - 1, & \begin{cases} 0, & i^g(\lambda) \notin \Gamma^g_{UW} \\ s, & i^g(\lambda) \in \Gamma^g_{UW} \end{cases} \end{cases} \tag{6}$$

and $\alpha = \{1, \dots, R\}$, $A_{UW} = \frac{a_{UW}}{b_{UW}}$, where a_{UW} and b_{UW} are the number of active subcarriers within a subblock and the subblock size for UW, respectively. As illustrated in Figure 1, after CP addition, R OFDM symbols of UW are concatenated to generate its time domain signal as

$$\mathbf{x}_{UW} = [\mathbf{x}^1_{UW}, \mathbf{x}^2_{UW}, \dots, \mathbf{x}^R_{UW}]_{1\times N}. \tag{7}$$

Lastly, $\mathbf{x}_{UN}$ and $\mathbf{x}_{UW}$ are superimposed before transmission as

$$\mathbf{x}_{tot} = \mathbf{x}_{UN} + \mathbf{x}_{UW}. \tag{8}$$

The superimposed signal $\mathbf{x}_{tot}$ passes through Rayleigh fading channel $h(n) = \sum_{l=0}^{L-1} h(l)\delta(n-l)$ with L Gaussian random variable taps following $h(l) \sim \mathcal{CN}(0, 1/L)$. The received signal is represented as

$$y_{tot}(n) = \sum_{l=0}^{L-1} h(l) x_{tot}(n-l) + w(n), \tag{9}$$

where $w(n) \sim \mathcal{CN}(0, \sigma^2)$ is additive white Gaussian noise (AWGN). After CP removal at the receiver side, each user perform FFT in the spectrum region where its data is transmitted. Later, log-likelihood ratio (LLR) detector is utilized for the estimation of the transmitted bits both by the indices of active subcarriers and by conventional Q-ary modulation [15].

3 | OFDM-IM WITH INI-AWARE SMS UTILIZATION

In the literature, it is shown that the INI level in an OFDM block varies with the subcarrier index, and UN is severely affected by INI due to the high side lobes of UW [5, 6]. Hence, in this study, the effect of INI in an OFDM-IM system is assessed and alleviated by the utilization of different SMSs for UN and UW.

In conventional OFDM-IM, COMB SMS is adopted for the mapping of information bits to subcarrier indices due to its low complexity, especially for high b values [15]. However, it is seen that COMB SMS gives a higher activation probability to the first subcarriers (FSs) that are located at the beginning of subblocks compared with the last subcarriers (LSs) located at the end, as shown in Figures 2 and 3 [22]. Conventionally, COMB SMS is utilized for the users who perform orthogonal transmission

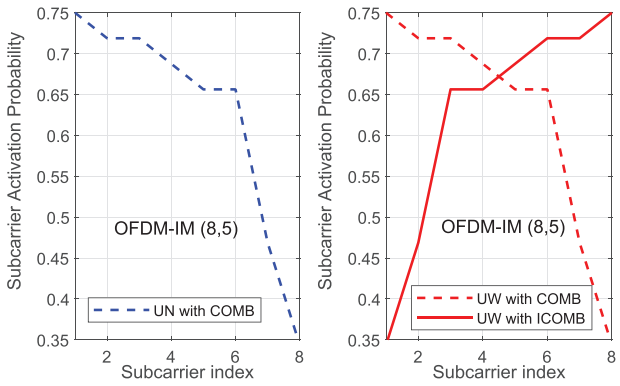


FIGURE 3 SAP for OFDM-IM with (8,5) in the case of COMB and ICOMB SMS utilization

[16]. However, under INI conditions, this causes an increase in the INI impact on the system. Considering Figures 2 and 3, the utilization of COMB SMS for UW will definitely lead to higher INI than that of conventional OFDM due to both higher activation probability of its FSs and the increased subcarrier power in OFDM-IM with the ratio of b/a. On the other hand, the utilization of COMB SMS for UN results in low INI on UW owing to the low activation probability of its LSs. As such, in this study, we propose a different SMS for each user in order to minimize the INI impact. Specifically, the conventional COMB is assigned for the UN as the resultant INI is low, while a modified version of COMB is proposed to be used at the UW aiming to minimize the INI impact at its end. In the following, the adopted SMS at each user is described in detail.

3.1 | Combinatorial SMS for UN

COMB SMS associates each possible index combination Γ with a single natural number Z_{UN} corresponding to an unique lexicographically ordered sequence as $J_{UN} = \{\gamma_{a_{UN}} \ \dots \ \gamma_1\}$ with $\gamma \in \{0, \dots \ b_{UN} - 1\}$ [15]. Z_{UN} can take value till $2^{\lfloor \log_2 C(b_{UN}, a_{UN}) \rfloor} \leq C(b_{UN}, a_{UN})$. After the mapping of incoming p_1 bits to a natural number Z, the elements of J_{UN} are given as follows

$$Z_{UN} = C(\gamma_{a_{UN}}, a_{UN}) \ + \ \dots \ + C(\gamma_2, 2) + C\ (\gamma_1, 1). \quad (10)$$

Later, the maximal γ_a component that satisfies $Z_{UN} \geq C(\gamma_{a_{UN}}, a_{UN})$ and then the maximal $\gamma_{a_{UN}-1}$ that satisfies $Z_{UN} - C(\gamma_{a_{UN}}, a_{UN}) \geq C(\gamma_{a_{UN}-1}, a_{UN} - 1)$ are chosen to decide on γ values. The process continues until $Z_{UN} - \sum_{i=2}^{a_{UN}} C(\gamma_i, i) \geq C(\gamma_1, 1)$. Once J_{UN} is generated, active subcarrier indices in COMB mapping are obtained as

$$\Gamma_{UN} = J_{UN} + 1. \quad (11)$$

3.2 | Inverse combinatorial SMS for UW

In this study, considering the SAP pattern of COMB mapping, the implementation of COMB scheme in a reverse manner is proposed for UW in order to control INI on the UN which is more affected by INI. Inverse COMB (ICOMB) SMS does not pose additional complexity, and after finding a specific J_{UW} for each Z_{UW} with COMB method, active subcarrier indices in ICOMB mapping are determined as

$$\Gamma_{UW} = b_{UW} - J_{UW}. \quad (12)$$

Note that when UN and UW are located in the inverse order with respect to Figure 1, ICOMB and COMB SMSs need to be used for UN and UW, respectively.

4 | INI ANALYSIS FOR OFDM-IM SYSTEMS

Considering the system model with two users, the power of INI caused by kth subcarrier of UN on k'th subcarrier of UW in OFDM systems is calculated as [6]

$$I_{OFDM}^{UW}(k, k') = \frac{\rho^{UN}(k)}{NM}\left(\frac{\left|\sin\left[\frac{\pi}{R}(1 + \chi R_{CP})\Delta k\right]\right|^2}{\left|\sin\left[\frac{\pi}{N}(\Delta k + N_R)\right]\right|^2}\right) \quad (13)$$

for $0 \leq k \leq N_R$, and $\{0 \leq k' \leq M_R - 1 \ : \ k'/R \in \mathbb{Z}\}$,

TABLE 1 Occurrence number of kth subcarrier considering the subcarrier combinations, start with γ subcarrier

γ	Subcarriers							
	$k \geq a$					k < a		
	$k = b$	$k = b-1$	$\cdots$	$k = a+1$	$k = a$	$k = a-1$	$\cdots$	$k = 1$
$\gamma = a$	×	×	$\cdots$	×	$C(a-1, a-1)$	$C(a-2, a-2)$	$\cdots$	$C(a-2, a-2)$
$\gamma = a+1$	×	×	$\cdots$	$C(a, a-1)$	$C(a-1, a-2)$	$C(a-1, a-2)$	$\cdots$	$C(a-1, a-2)$
$\cdots$	×	×	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\gamma = b-1$	×	$C(b-2, a-1)$	$\cdots$	$C(b-3, a-2)$	$C(b-3, a-2)$	$C(b-3, a-2)$	$\cdots$	$C(b-3, a-2)$
$\gamma = b$	$C(b-1, a-1)$	$C(b-2, a-2)$	$\cdots$	$C(b-2, a-2)$	$C(b-2, a-2)$	$C(b-2, a-2)$	$\cdots$	$C(b-2, a-2)$
$\mathcal{O}(k)$	$C(\gamma-1, a-1) + \sum_{i=\gamma+1}^{b} C(i-2, a-2)$					$\sum_{i=a}^{b} C(i-2, a-2)$		

while the INI power caused by kth subcarrier of UW on k'th subcarrier of UN is

$$I_{OFDM}^{UN}(k,k') = \frac{\rho^{UW}(k)}{NM}\left(\frac{\left|\sin\left[\frac{\pi}{R}(1+\chi R_{CP})\Delta k\right]\right|^2}{\left|\sin\left[\frac{\pi}{N}(\Delta k+N_R)\right]\right|^2} - \frac{\chi\left|\sin\left[\frac{\pi}{R}(1+R_{CP})\Delta k\right]\right|^2}{\left|\sin\left[\frac{\pi}{N}(\Delta k+N_R)\right]\right|^2}\right) \quad (14)$$

for $0 \le k \le M_R$ and $\{0 \le k' \le N_R - 1 \,:\, k'/R \in \mathbb{Z}\}$,

where $\chi = (1-R)$, and $\Delta k = (k-k')$. $\rho^{UN}(k)$ and $\rho^{UW}(k)$ refer to the subcarrier power for UN and UW, respectively. R_{CP} is the CP ratio that provides CP size larger than the channel spread. Since not all the subcarriers are utilized in OFDM-IM systems, the effect of both subcarrier power increase due to partial activation ratio and the probability of kth subcarrier being active $P(X_{IM}(k) \neq 0)$ need to be considered to calculate the exact INI power. In this regard, in OFDM-IM systems with multi-numerology, $I_{IM}^{UW}(k,k')$ and $I_{IM}^{UN}(k,k')$ are calculated as

$$I_{IM}^{UW}(k,k') = \frac{1}{\mathcal{A}_{UN}} I_{OFDM}^{UW}(k,k') P(X_{IM}^{UN}(k) \neq 0), \quad (15)$$

and

$$I_{IM}^{UN}(k,k') = \frac{1}{\mathcal{A}_{UW}} I_{OFDM}^{UN}(k,k') P(X_{IM}^{UW}(k) \neq 0), \quad (16)$$

respectively. $P(X_{IM}^{UN}(k) \neq 0)$ and $P(X_{IM}^{UW}(k) \neq 0)$ refer to activation probability of kth subcarrier for UN and UW, respectively, which are assessed in the following subsection in detail. Note that $\mathcal{A}_{UN/UW} = 1$ and $P(X_{IM}^{UN/UW}(k) \neq 0) = 1$ for OFDM systems, since $a_{UN/UW} = b_{UN/UW}$.

4.1 | Activation probability of kth subcarrier in OFDM-IM

In this subsection, the activation probability of kth subcarrier in OFDM-IM is provided for COMB and ICOMB SMSs in order to calculate which activation ratio can provide lower INI level compared to the OFDM case for both UN and UW.

For a given (b,a) pair, the total number of possible subcarrier combinations is $N_\Gamma = C(b,a)$ when data transmission is performed by their full permutations [24]. Considering the descending order of active subcarrier indices in a given combination Γ_γ, the first subcarrier (γ) can take a value between $\gamma = \{a, a+1, \dots, b\}$. Thus, N_Γ combinations can be divided into $N_C = b - a + 1$ categories considering the possible γ values. To exemplify, the combinations can start with $\gamma = \{3,4,5,6,7,8\}$ for OFDM-IM with (8,3) and $N_C = 6$. Table 1 illustrates the occurrence number of kth subcarrier in $N_{\Gamma_\gamma} = C(\gamma-1, a-1)$ combinations that start with a certain value of γ. In this regard, the activation probability of kth subcarrier when full permutations of subcarrier combinations are utilized can be generalized as

$$P(X(k) \neq 0) = \frac{1}{N_\Gamma} O(k), \quad (17)$$

where

$$O(k) = \begin{cases} \sum_{i=a}^{b} C(i-2, a-2), & k < a \\ C(\gamma-1, a-1) + & \\ \sum_{i=\gamma+1}^{b} C(i-2, a-2), & k \ge a. \end{cases} \quad (18)$$

ALGORITHM 1 Calculation of the occurrence number of kth subcarrier for $\gamma = z$

$r_2 = 0, N_{comb} = 0, i = 1, l = 1, j = a$ ▷ Initialization
$O = [0\ 0\ \dots\ 0]_{1\times b}$ ▷ $O_{IM_z}(k)$
while $N_{comb} < N_{\gamma_{IM}}$ **do**
  $N_{comb} = N_{comb} + C(j-1, a-1)$
  $j = j + 1$
end while
$z = j$ ▷ $Z \ge N_{\Gamma_{IM}}$
$r_1 = N_{\gamma_{IM}} - \sum_{i=a}^{z-1} C(i-1, a-1)$
k = z
while $r_2 \neq r_1$ **do**
  if $r_1 < C(k-1, a-i)$ **then**
    $O(k) = O(k) + r_1$
  else if $r_1 > C(k-1, a-i)$ **then**
    while $r_2 \neq r_1$ **do**
      if $C(k-1, a-i) \le r_1 - r_2$ **then**
        $O(k) = O(k) + C(k-1, a-i)$,
        $O(k-1:1) = O(k-1:1) + C(k-2, a-i-1)$
        $r_2 = r_2 + C(k-1, a-i)$
        $k = k - 1$
      else
        $O(k:k-1) = O(k:k-1) + r_1 - r_2$
        $O(k-2:1) = O(k-2:1) + C(k-2, a-i-1)$
        $r_2 = r_1$
      end if
    end while
  end if
  $k = k - 1$
  $i = i + 1$
end while

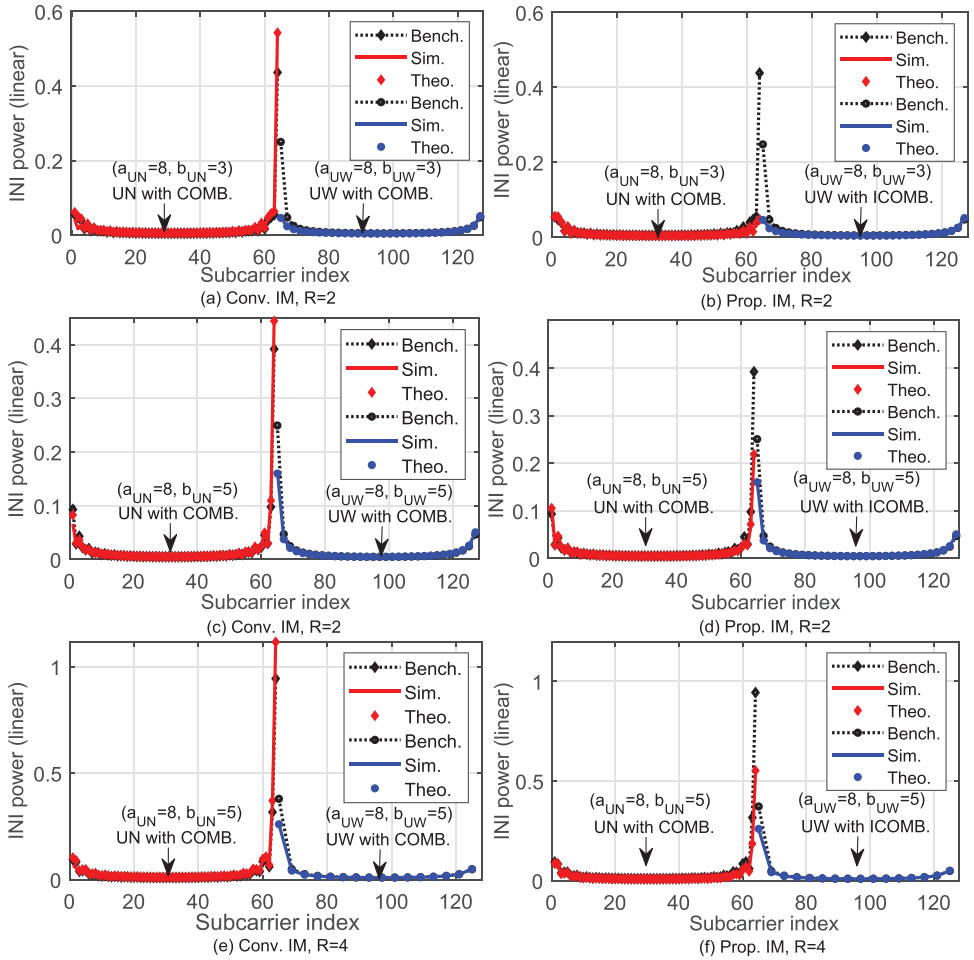


FIGURE 4 INI analysis in OFDM, conventional OFDM-IM and the proposed OFDM-IM for UN and UW considering different SARs and R values

As an example, if the combination starts with $\gamma = b = 8$ for OFDM-IM with (8,3), the activation probability of bth and the remaining subcarriers can be calculated as $C(7,2)$ and $C(6,1)$, respectively. The summation of the number of subcarrier combinations N_{Γ_γ} for each γ value should be equal N_Γ as follows

$$N_\Gamma = \sum_{\gamma=a}^{b} N_{\Gamma_\gamma} = \sum_{\gamma=a}^{b} C(\gamma - 1, a - 1). \tag{19}$$

However, OFDM-IM utilizes $N_{\Gamma_{IM}} = 2^{p_1}$ combinations out of N_Γ to transmit the index bits, given that $N_{\Gamma_{IM}} \leq N_\Gamma$ [25]. COMB SMS first utilize the subcarrier combinations start with $\gamma = a$ and then continues up to $\gamma = b$. In other words, as given in (10), Z can take value between from 0 to $N_{\Gamma_{IM}} - 1$ and thus OFDM-IM transmission do not utilize the subcarrier combinations that are associated with $Z \geq N_{\Gamma_{IM}}$. Considering γ_ζ that is the γ value in which Z becomes greater than $N_{\Gamma_{IM}}$, only r_1 combinations out of $C(\gamma_\zeta - 1, a - 1)$ are used. For instance, $N_{\Gamma_{IM}} = 32$ and $\gamma = \{3, 4, 5, 6, 7\}$ for OFDM-IM with (8,3), and only $r_1 = 12 < C(6,2)$ combinations are used for $\gamma_\zeta = \gamma = 7$.

Algorithm 1 provides the calculation of number of times kth subcarrier is utilized in case of $\gamma = \gamma_\zeta$ for any (b, a) values. In this regard, SAP for OFDM-IM is achieved as

$$P_{IM}(X(k) \neq 0) = \frac{1}{N_{\Gamma_{IM}}} O_{IM}(k) = \frac{1}{N_{\Gamma_{IM}}} \sum_{\gamma=a}^{b} O_{IM_\gamma}(k), \tag{20}$$

where

$$O_{IM_\gamma}(k) = \begin{cases} \text{if } \gamma > \gamma_\zeta, \ 0 \\ \text{if } \gamma = \gamma_\zeta, \ O_{IM_{\gamma_\zeta}}(k) \text{ as in Algorithm 1} \\ \text{if } \gamma < \gamma_\zeta, \begin{cases} \sum_{i=a}^{\gamma_\zeta - 1} C(i-2, a-2), \ k < a \\ C(\gamma - 1, a - 1) + \\ \sum_{i=\gamma+1}^{\gamma_\zeta - 1} C(i-2, a-2), \ k \geq a. \end{cases} \end{cases} \tag{21}$$

It is important to mention that $\sum_{\gamma=a}^{b} O_{IM_\gamma}(k) = \sum_{\gamma=a}^{\gamma_\zeta} O_{IM_\gamma}(k)$, since $O_{IM_\gamma}(k) = 0$ for $\gamma > \gamma_\zeta$. Since ICOMB SMS works in an

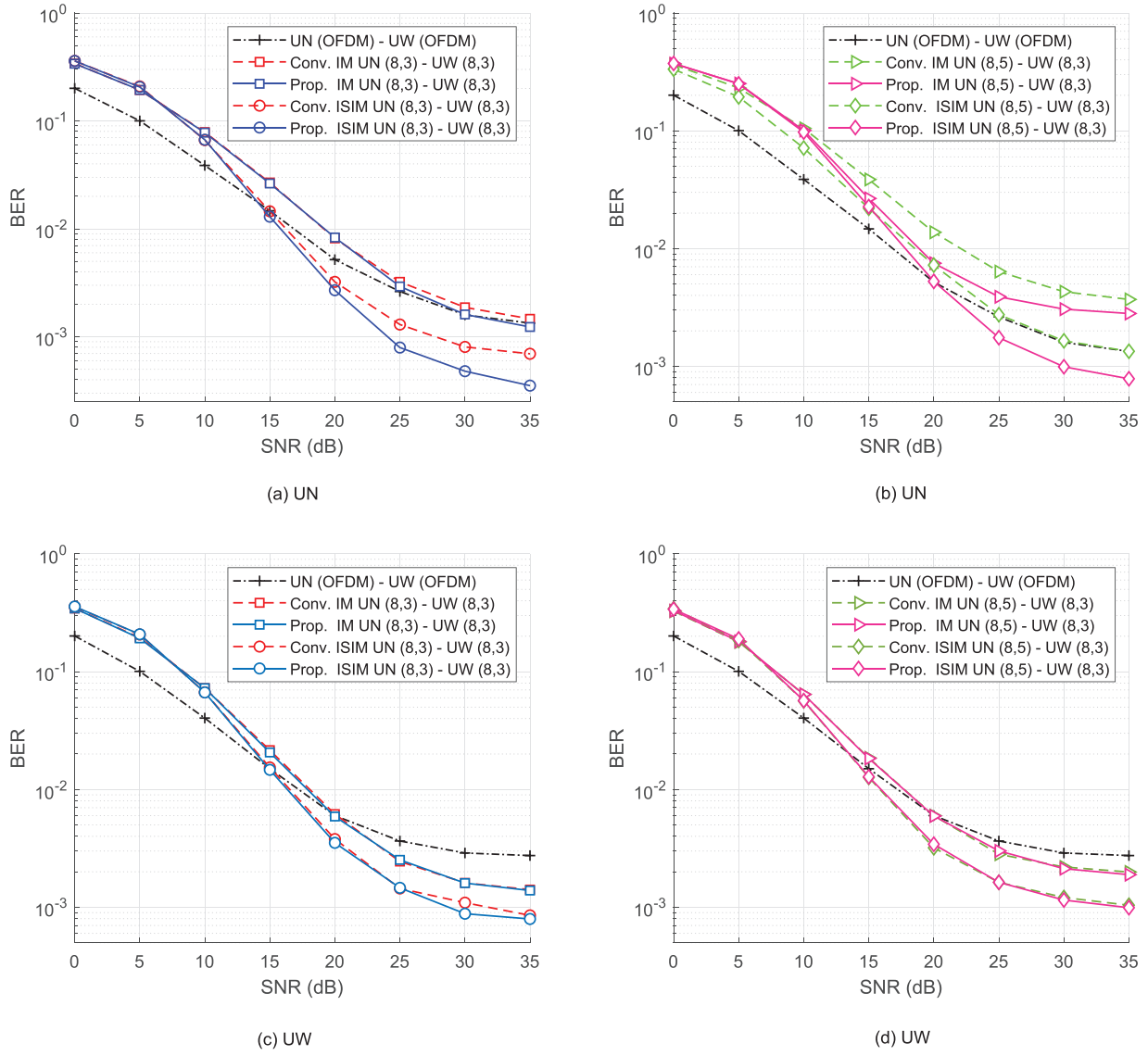


FIGURE 5 BER performances of OFDM and OFDM-IM systems with fixed and INI-aware SMS utilization considering different SARs, while $R = 2$

inverse manner of COMB SMS, SAPs for OFDM-IM with ICOMB corresponds to

$$P_{IM}^{ICOMB}(X(b-k+1) \neq 0) = P_{IM}(X(k) \neq 0). \quad (22)$$

5 | NUMERICAL AND SIMULATION RESULTS

In this section, Monte Carlo simulations are performed to analyze the INI effect in OFDM-IM systems considering fixed and INI-aware SMSs utilization for UN and UW. The conventional OFDM is adopted as a benchmark. The obtained results are validated via theoretical INI analysis. $N = 128$, $R_{CP} = 1/4$, and binary phase-shift keying (BPSK) modulation are used in all simulations. Therefore, $M = 64$ and $M = 32$ are selected for $R = 2$ and $R = 4$, respectively. In order to avoid channel impairments, it is assumed that $L = 10$, and $L = 5$ for $R = 2$ and $R = 4$, respectively. In the figures, conventional OFDM-IM with multi-numerology (Conv. IM/ISIM) corresponds to the utilization of COMB SMS for both users, and the proposed OFDM-IM with multi-numerology (Prop. IM/ISIM) uses COMB SMS for UN and ICOMB SMS for UW. Moreover, OFDM-IM (8,3) provides the same spectral efficiency as OFDM, while OFDM-IM (8,5) offers a 25% increase in data rate [15].

Figure 4 illustrates the INI power levels in OFDM, Conv. IM, and Prop. IM in case of $R = 2$ and $R = 4$. As illustrated in Figure 4a,c, and e, direct utilization of OFDM-IM for the coexistence of multi-numerology leads to the higher INI levels for UN and the lower INI levels for UW with respect to OFDM. This is due to the subcarrier activation pattern of COMB SMS illustrated in Figure 2. For instance, in the case of UN with OFDM-IM (8,3), the last subcarrier is not used for data

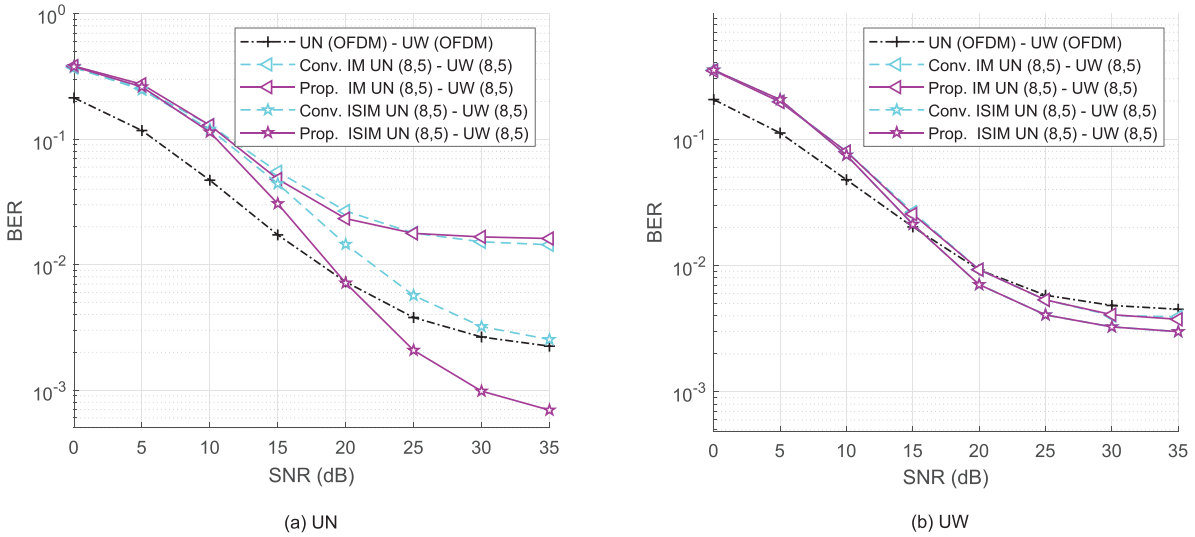


FIGURE 6 BER performances of OFDM and OFDM-IM systems with fixed and INI-aware SMS utilization considering different SARs, while $R = 4$

transmission and consequently, the INI level is significantly reduced on UW. On the other hand, the first subcarrier of UW with OFDM-IM (8,3) is utilized with higher activation probability, and its power is $8/3$ times greater than that of UW with OFDM due to partial subcarrier activation. By means of ICOMB SMS, it is aimed to utilize the subcarriers of UW near to UN with low activation probability to avoid high side-lobe levels. The utilization of INI-aware SMSs in OFDM-IM provides the control of INI for the UN as seen in Figure. 4b,d, and f. Since the UN employs COMB mapping in both conventional OFDM-IM implementation (Figure 4a,c, ande) and the proposed OFDM-IM implementation (Figure. 4b,d, and f), INI level on UW caused by UN is the same. It is worth to mention that OFDM-IM with (8,3) has the same spectral efficiency with conventional OFDM, while OFDM-IM with (8,5) offers higher spectral efficiency. Although direct utilization of OFDM-IM in multi-numerology systems leads to increase on INI level for UN, it significantly reduces INI level for UW under the same spectral efficiency as shown in blue lines in Figure 4a,b. Moreover, UW still faces with less INI in OFDM-IM while providing higher spectral efficiency compared to conventional OFDM, as shown in Figure 4c–f. The INI level can be further reduced with different activation ratios. As a result, it is important to note that the active subcarrier position plays a key role in OFDM-IM systems to alleviate INI, rather than SAR, due to its high power level.

Figures 5 and 6 present the BER performances for UN and UW considering different SARs and R values. Classical OFDM-IM is severely affected by INI due to its delicate receiver against interference, which is inevitable in practice. Fortunately, OFDM-IM provides diversity gain with frequency domain interleaving known as OFDM-ISIM. As seen in Figures 5a,b and 6a, for UN, both conventional and the proposed OFDM-IM cannot overcome the performance of OFDM, since high interference causes the wrong detection of active subcarrier indices resulting in error propagation. With the aid of diversity gain, OFDM-ISIM provides better performance than conventional OFDM and OFDM-IM under INI impact. Additionally, it reveals the superiority of INI-aware SMS utilization in IM-based OFDM systems. To exemplify, in Figure 6a, Conv. ISIM offers similar performance with OFDM, while Prop. ISIM provides significant improvement in error rate. It is observed that ICOMB SMS utilization for UW becomes more substantial to improve the error performance under high INI levels ($R = 4$). On the other hand, as in Figures 5c,d and 6b, IM utilization in UN directly provides better performances for UW than that of OFDM, regardless of the utilized SMS. Consequently, the flexible structure of OFDM-IM provides opportunities to control INI for UN and UW.

6 | CONCLUDING REMARKS

Here, INI analysis is performed for OFDM-IM systems, and an INI-aware SMS is introduced to alleviate its impact. It is shown that conventional OFDM-IM is vulnerable to INI and can lead to worse performance than OFDM for UN. As the ratio between adjacent numerologies increases, the need for flexible frame design becomes imperative for OFDM systems in order to combat INI. The adaptive structure of OFDM-IM allows to control of INI and consequently improving the BER performance via the proposed SMS. It is expected that the joint utilization and optimization of the INI-aware SMS with well-known INI reduction techniques, such as, guard band and filtering, can further reduce INI levels with moderate loss of spectral efficiency, left as future work.

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REFERENCES

1. Dang, S., et al.: What should 6G be? Nat. Electron. 3(1), 20–29 (2020)
2. Yazar, A., Dogan-Tusha, S., Arslan, H.: 6G vision: An ultra-flexible radio access technology perspective. arXiv:200907597, pp. 1–15 (2020)
3. 3GPP: 5G NR: Physical channels and modulation. ETSI TS 138 211 V1530, pp. 1–98 (2018)
4. 3GPP: IMT Vision - Framework and overall objectives of the future development of IMT for 2020 and beyond. Recommendation ITU-R M2083-0, pp. 1–21 (2015)
5. Zhang, X., et al.: Mixed numerologies interference analysis and inter-numerology interference cancellation for windowed OFDM systems. IEEE Trans. Veh. Technol. 67(8), 7047–7061 (2018)
6. Kihero, A.B., Solaija, M.S.J., Arslan, H.: Inter-numerology interference for beyond 5G. IEEE Access 7(10), 146512–146523 (2019)
7. Memisoglu, E., et al.: Guard band reduction for 5G and beyond multiple numerologies. IEEE Commun. Lett. 24(3), 644–647 (2020)
8. Cheng, X., et al.: Inter-numerology interference analysis and cancellation for massive MIMO-OFDM downlink systems. IEEE Access 7(12), 177164–177176 (2019)
9. Choi, J., et al.: A transceiver design for spectrum sharing in mixed numerology environments. IEEE Trans. Wirel. Commun. 18(5), 2707–2721 (2019)
10. Mao, J., et al.: Interference analysis and power allocation in the presence of mixed numerologies. IEEE Trans. Wirel. Commun. 19(8), 5188–5203 (2020)
11. Basar, E., et al.: Index modulation techniques for next-generation wireless networks. IEEE Access 5(8), 16693–16746 (2017)
12. Dogan-Tusha, S., et al.: Multidimensional index modulation for 5G and beyond wireless networks. Proc. IEEE 109(2), 170–199 (2021)
13. Dogan, S., Tusha, A., Arslan, H.: NOMA with index modulation for uplink URLLC through grant-free access. IEEE J. Sel. Topics Signal Process. 13(6), 1249–1257 (2019)
14. Althunibat, S., Mesleh, R., Qaraqe, K.A.: IM-OFDMA: A novel spectral efficient uplink multiple access based on index modulation. IEEE Trans. Veh. Technol. 68(10), 10315–10319 (2019)
15. Basar, E. et al.: Orthogonal frequency division multiplexing with index modulation. IEEE Trans. Signal Process. 61(22), 5536–5549 (2013)
16. Dogan, S., Tusha, A., Arslan, H.: OFDM with index modulation for asynchronous mMTC networks. Sensors 18(4), 1280–1295 (2018)
17. Tusha, A., Dogan, S., Arslan, H.: IQI mitigation for narrowband IoT systems with OFDM-IM. IEEE Access 6, 44626–44634 (2018)
18. Tusha, A., Dogan, S., Arslan, H.: Performance analysis of frequency domain IM schemes under CFO and IQ Imbalance. In: IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC), pp. 1–5 (2019)
19. Ma, Q., et al.: Error probability analysis of OFDM-IM with carrier frequency offset. IEEE Commun. Lett. 20(12), 2434–2437 (2016)
20. Jo, H., et al.: A technique to reduce PAPR for OFDM-IM using multiple mapping rules for IM. In: IEEE Vehicular Technology Conference (VTC), pp. 1–5 (2018)
21. Chandran, K.S., Ali, C.K.: Filtered-OFDM with index modulation for mixed numerology transmissions. In: International Conference on Advanced Computing and Communication Systems (ICACCS), pp. 306–310 (2020)
22. Wen, M., et al.: Equiprobable subcarrier activation method for OFDM with index modulation. IEEE Commun. Lett. 20(12), 2386–2389 (2016)
23. Xiao, Y., et al.: OFDM with interleaved subcarrier-index modulation. IEEE Commun. Lett. 18(8), 1447–1450 (2014)
24. Wen, M., et al.: Multiple-mode orthogonal frequency division multiplexing With index modulation. IEEE Trans. Commun. 65(9), 3892–3906 (2017)
25. Wen, M., et al.: On the achievable rate of OFDM with index modulation. IEEE Trans. Signal Process. 64(8), 1919–1932 (2016)

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