Design criteria for Herriott-type multi-pass cavities for ultrashort pulse lasers

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Abstract: We investigate the general characteristics of Herriott-type multi-pass cavities (MPC) for femtosecond lasers. MPCs can be used to increase the laser pulse energy by extending the laser cavity path length and decreasing the repetition rate, as well as to make standard repetition rate lasers more compact. We present an analytical design condition for MPCs which preserve the Gaussian beam q parameter, enabling the laser path length to be extended while leaving the Kerr-lens modelocking operating point of the cavity invariant. As a specific example, we analyze q preserving MPCs consisting of a flat and curved mirror to obtain analytical expressions for the cavity length. This predicts the optimum MPC designs that minimize the pulse repetition rate for given specifications. These design conditions should prove useful for designing a wide range of high pulse energy or compact femtosecond lasers.

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Herriott-type multi-pass cavities (MPC) can be used in a wide range of femtosecond laser designs. MPCs enable the laser pulse energy to be increased by extending the cavity path length and decreasing the repetition rate. MPCs also enable the development of very compact lasers while preserving standard repetition rates and pulse energies. In its simplest form, an MPC consists of a stable two-mirror resonator and a mechanism for injecting and extracting light beams. When the MPC parameters are properly adjusted, the incident beam injected with the correct offset and tilt, undergoes multiple bounces before exiting. The successive bounces of the beam viewed in a given reference plane (for example on one of the end mirrors) form an elliptical or circular spot pattern. Using appropriate design conditions, the MPC can leave the Gaussian beam q parameter invariant. This means that diffractive beam spreading effects are exactly cancelled as a result of the periodic focusing inside the cavity. First introduced by Herriott et al [1,2], MPCs have long been used in many applications such as accurate optical loss measurements [2], stimulated Raman scattering [3,4], long-path absorption spectroscopy [5], and high-speed path-length scanning [6].

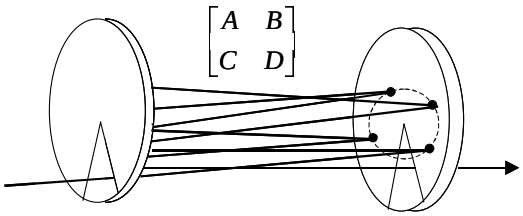


Fig. 1. A schematic of a general multi-pass cavity. One round trip is represented by the ABCD matrix.

Previous studies have demonstrated improvements in laser pulse energy using low repetition rate femtosecond lasers created by extending the laser cavity with relaying imaging [7]. Recently high pulse energy, KLM femtosecond lasers using MPCs for extending the cavity length have been demonstrated [8,9]. Increasing the overall cavity path length lowers the pulse repetition rate while maintaining the same average output power and thereby increases the output pulse energy. Furthermore, since amplifiers or cavity dumpers are not required, the system design is simplified and the overall cost is significantly reduced. These laser sources are attractive in many applications such as pump-probe studies where high-energy pulses at reduced average powers obviate undesirable effects such as thermal loading. In addition, the high-energy output of an MPC-type femtosecond laser may also be useful for seeding high energy amplifiers because higher initial pulse energies will require lower amplifier gain, reducing the amplification of the unwanted pedestals and improving the contrast of the amplified pulses.

Several issues need to be addressed in the practical design of MPCs for high-energy ultrafast lasers. In general, the cavity operating point is very important for Kerr lens modelocked operation and modelocking is only possible in a small subset of the cavity stability region. If the cavity repetition rate is increased by extending the length of a standard four-mirror cavity, the cavity operating point, including the stability region and spot sizes change dramatically. This makes re-establishing KLM operation very difficult. These problems are solved by keeping the q parameter invariant. In addition, even if the pulse energy changes as a result of using an MPC, keeping the same q parameter operating point is important if the laser is modelocked in the soliton pulse regime. In this case, the cavity dispersion is proportionally scaled up as the pulse energy is increased to enable the generation

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of pulses with approximately the same duration. For these reasons, maintaining the same q parameter is desirable. This is guaranteed if the MPC is designed in such a way that the exiting beam has the same q parameter as the beam incident on the MPC. In what follows, we will refer to this as the "q preserving" configuration of the MPC. For the design of MPC femtosecond lasers, it is helpful to have general guidelines on how to construct q preserving MPCs. In addition, it is also important to know how the optical path length and the repetition rate vary for different "q preserving" configurations in order to maximize the output energy of the oscillator.

In this paper, we present a general analysis of MPCs that can be used for designing high pulse energy or compact femtosecond lasers. We first consider an arbitrary MPC and derive the general condition required to preserve the q parameter of the Gaussian beam. This condition can be related to the spot pattern that the beam forms upon successive transits inside the MPC. We prove that the MPC is q preserving when the angular separation of successive bounces is π times a rational number. This leads, in principle, to infinitely many possible q-preserving configurations. As an example of a typical configuration, we analyze an MPC consisting of a flat and a curved mirror. The pulse repetition rates for the corresponding mirror separations of q preserving MPC configurations are calculated. Finally, we calculate the optimum spot patterns that give the minimum achievable pulse repetition rate, or the longest cavity path length, for a given MPC configuration.

Let us begin by considering the general multi-pass cavity (MPC) shown in Fig. 1. The ray transfer matrix $M_{T,}$ which represents one round trip, is given by

$$M_T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}. \tag{1}$$

Because the ray necessarily comes back to the same region in the cavity after each round trip, the determinant of M_T is unity. Let the initial ray be represented by a vector $\vec{r}_i$ given by

$$\vec{r}_i = \begin{bmatrix} r_0 \\ r_0' \end{bmatrix}. \tag{2}$$

In the above, r_0 and r_0' give the initial ray displacement from the optical axis and the initial inclination of the ray, respectively. We also note in passing that in typical MPC designs, the angles of inclination are small and radii of curvature of the mirrors very large. Therefore, astigmatic effects arising from tilted beam incidence can be neglected. After n round trips, the ray vector $\vec{r}_n$ becomes

$$\vec{r}_n = M_T^n \vec{r}_i. \tag{3}$$

The stability of the cavity, in other words, the condition that the rays remain confined to the optical axis after an arbitrary number of round trips, is guaranteed when A and D satisfy the condition

$$\left| \frac{A+D}{2} \right| \le 1. \tag{4}$$

In this case, the eigenvalues $\lambda_{1,2}$ and the associated eigenvectors $\vec{v}_{1,2}$ of M_T can be expressed as

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$$\lambda_1 = e^{i\theta},\ \ \vec{v}_1 = \frac{1}{B(B-C)}\begin{bmatrix} B \\ -\left(A - e^{i\theta}\right) \end{bmatrix}$$
$$\lambda_2 = e^{-i\theta},\ \ \vec{v}_2 = \frac{1}{B(B-C)}\begin{bmatrix} B \\ -\left(A - e^{-i\theta}\right) \end{bmatrix} \quad (5)$$

In Eq. (5), $\cos\theta = \frac{A+D}{2}$. The stability condition can be understood by noting that if Eq. (4) is not satisfied, then the magnitudes of the eigenvalues in Eq. (5) can be larger than 1, and the ray no longer remains confined to the optical cavity after multiple bounces. By using the well-known techniques of matrix algebra, it can be shown that M_T^n becomes

$$M_T^n = \begin{bmatrix} \frac{A-D}{2}\frac{\sin n\theta}{\sin\theta} + \cos n\theta & B\frac{\sin n\theta}{\sin\theta} \\ C\frac{\sin n\theta}{\sin\theta} & \frac{D-A}{2}\frac{\sin n\theta}{\sin\theta} + \cos n\theta \end{bmatrix}. \quad (6)$$

We can give a physical interpretation for the angle θ appearing in Eqs. (5) and (6) by considering the effect of the MPC on an incident ray with non-zero off-set and inclination along the x- and y- directions. By using Eq. (6), the transverse displacements x_n and y_n of the ray after n round trips can be expressed as

$$x_n = x_0\cos n\theta + \left(\frac{x_o(A-D) + 2Bx_0'}{2\sin\theta}\right)\sin n\theta$$
$$y_n = y_0\cos n\theta + \left(\frac{y_o(A-D) + 2By_0'}{2\sin\theta}\right)\sin n\theta \quad (7)$$

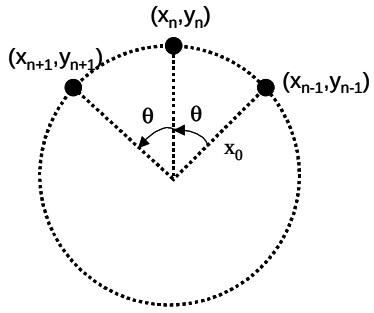


Fig. 2. The spot pattern formed by the beams upon successive transits will in general be elliptical at a given reference plane in the cavity. A circular spot pattern is obtained at the input reference plane when the position and tilt of the incident ray is adjusted according to Eq. 8.

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where (x_0, x_0') and (y_0, y_0') are the initial off-set and inclination in the x- and y-directions, respectively, at the input reference plane. If the initial ray parameters are adjusted so that

$$y_0 = 0$$
$$y_0' = \frac{x_0 \sin\theta}{B} \quad , \tag{8}$$
$$x_0' = \frac{x_0}{2B}(D - A)$$

then the ray describes a circle of radius x_0. Furthermore, θ corresponds to the change in angular position of the spots around the circle formed by successive bounces of the beam after each round trip. The resulting circular spot pattern at the location of the reference plane is schematically shown in Fig. 2.

We next consider the propagation of a Gaussian beam through the MPC. In order for the MPC to be q preserving, M_T^n must be $\pm I$, where *I* is the unity matrix. In the general case, we note from Eq. (6) that $M_T^n = (-1)^m I$ whenever

$$n\theta = m\pi \, . \tag{9}$$

Here, n and m are any two integers. Equation (9) summarizes the most important design rule for the construction of q preserving MPCs. Stated in words, whenever the angle, θ between successive bounces of the beam is π times a rational number, the ratio of two integers (*m/n*), the Gaussian q parameter remains invariant after n round trips. Note that m gives the number of semicircular arcs that the bouncing beam traverses on one of the mirrors before the q parameter is transformed back to its initial value. For each value of m, every integer value of n corresponds to a q preserving configuration of the MPC. When m is even, the bouncing beam traverses an integral number of full circular trajectories and comes back to the initial entry position before exiting the MPC. This q-preserving case is identical with Herriott's "re-entrant" condition discussed in Ref. [1].

As an example of a set of possible solutions to Eq. (9), consider a cavity where the beam makes n=9 round trips. Figure 3 shows examples of different spot patterns (indicated by solid circles), which produce q preserving configurations. The cases with m = 1, 2, 3, 4, and 5 are shown. The corresponding angular advance θ between successive bounces is mπ/9. The value of θ is indicated in each figure. Note that open circles are equally spaced on a full circle with an angular separation of 2π/9 while solid circles are separated by θ. The arrow further indicates the direction in which the spot pattern evolves after successive round trips. The integer next to each solid circle designates the number of round trips that the beam has undergone before reaching the reference plane.

For example, Fig 3(a) with m=1 and n=9, corresponds to a spot pattern where the 9th bounce is on the opposite side of the mirror, or at an angle of π away, from the initial beam. In Fig 3(b) with m=2 and n=9, the 9th bounce in the spot pattern is at the same position as the initial beam. The beam has bounced in a circular pattern with a net angular sweep of 2π around the circle. In Fig 3(c) with m=3 and n=9, the 9th bounce overlaps with the 3rd bounce. This design cannot be used in practice because it is impossible to extract the beam after 9 bounces without blocking it during earlier bounces. Figure 3(d) with m=4 and n=9 is analogous to the case when m=2, where the 9th bounce is at the same position as the initial beam, except that the beam has bounced with a net angular sweep of 4π around the circle.

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Finally, Fig 3(e), with m=5 and n=9, shows a case where the 9th bounce is again on the opposite side of the mirror from the initial beam. In this case, the beam has bounced in a circular pattern with a net angular sweep of 5π. Additional cases for other values of m are not shown, but can easily be constructed by extending these results.

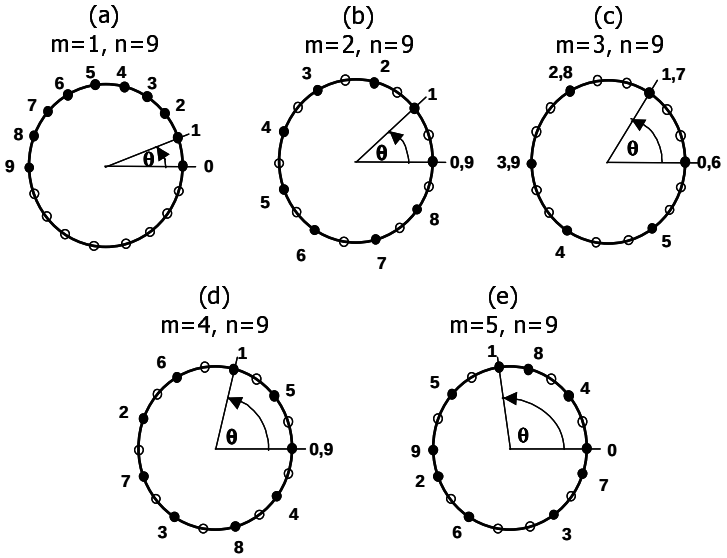


Fig. 3. Spot patterns corresponding some possible q preserving configurations for the case where m = 1,2,3,4,5 and n=9.

The general results derived above can also be examined for the specific case of an MPC consisting of a curved (M1, Radius of curvature=R) and a flat (M2) high reflector as schematically shown in Fig. 4. The separation between the MPC mirrors is L_0. A small flat mirror (M3) injects the incident beam into the MPC. A total of n round trips are completed when the beam is extracted from the MPC following a reflection from a curved pick-up mirror (M4) with the same radius of curvature as that of M1. For this particular case, we investigate the effect of the spot pattern on the repetition rate of the MPC. We choose the input reference plane z_{R1} of the MPC to be located at the position of the flat mirror M2 (See Fig. 4).

The ray transfer matrix M_T, representing one round trip starting at the input reference plane, becomes

$$M_T = \begin{bmatrix} 1-\dfrac{2L_0}{R} & 2L_0\left(1-\dfrac{L_0}{R}\right) \\ -\dfrac{2}{R} & 1-\dfrac{2L}{R} \end{bmatrix}, \tag{10}$$

with

$$\cos\theta = 1-\frac{2L_0}{R}. \tag{11}$$

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By varying L_0, we can find the possible values of θ that satisfy the q-preserving condition in Eq. (9).

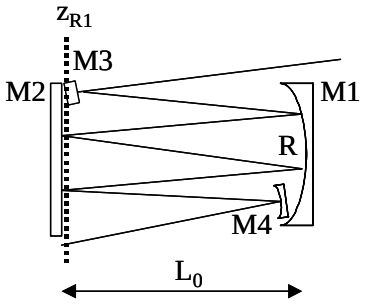


Fig. 4. A sketch of the MPC consisting of a flat and a curved mirror with radius R or focal length f=R/2, separated by a distance L0. This cavity can be analyzed to calculate the optimum configuration which minimizes the repetition rate or maximizes the beam path length for a given cavity size.

This simple MPC design permits the derivation of closed form expressions for the mirror separation and the pulse repetition rate. By using these results, optimum spot patterns that minimize the pulse repetition rate can be calculated. Note that the optical path length introduced by the MPC is simply $4nL_0$, giving a repetition frequency f_{rep} of

$$f_{rep} = \frac{c}{2nR\left(1-\cos\left(\frac{m\pi}{n}\right)\right)}, \tag{12}$$

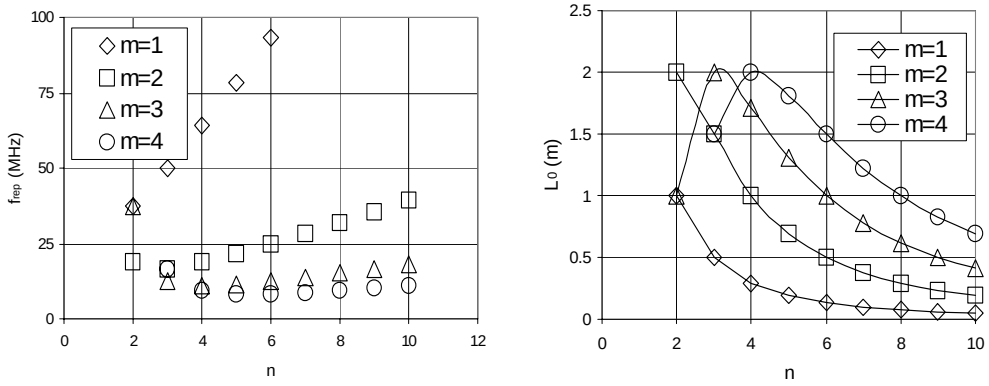


Fig. 5. Calculated variation of (a) the pulse repetition rate and (b) the corresponding mirror separation for the q preserving configurations of the multi-pass cavity for different values of m for the flat – curved MPC case.

for the MPC when additional arm lengths of the resonator are neglected. In Eq. (12), c is the speed of light. The q preserving condition in Eq. (9) was used to obtain Eq. (12). In practical femtosecond lasers, the overall repetition rate will be smaller than f_{rep} given in Eq. (12) due to

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the additional length of the KLM resonator. Figures 5(a) and (b) show the variation of f_{rep} and the corresponding mirror separation L_0 as a function of n for the case where R=2 meters. As noted earlier, each integer value of n (shown by markers of different shapes in Figs. 5 (a) and (b)) gives a q preserving configuration. Note that for each value of m, there is an optimum value n_{opt} of n, independent of R, which gives the largest possible optical length and the lowest repetition rate.

For example, in the case of m=2, n_{opt}=3 and the lowest possible repetition rate that can be obtained is c/9R. In the general case, n_{opt} is the integer closest to the exact solution of the transcendental equation

$$1 = \cos\left(\frac{m\pi}{n_{opt}}\right) + \frac{m\pi}{n_{opt}}\sin\left(\frac{m\pi}{n_{opt}}\right). \tag{13}$$

Table 1 gives the values of n_{opt}, the lowest possible repetition rate f_{min}, and the corresponding MPC separation L_{opt} for different values of m. These equations are helpful for general laser design because they give the lowest possible repetition rate or longest path length that can be built with a given q preserving MPC configuration.

Table 1. Calculated values of n_{opt}, the achievable minimum repetition rate f_{min}, and the corresponding MPC separation L_{opt} for different values of m for the flat-curved MPC.

m	1	2	3	4
n_{opt}	2	3	4	5
f_{min}	$\frac{c}{4R}$	$\frac{c}{9R}$	$\frac{c}{13.66R}$	$\frac{c}{18.09R}$
L_{opt}	$0.5R$	$0.75R$	$0.85R$	$0.90R$

In conclusion, we have presented a generalized analysis of multi-pass cavities that can be used for high pulse energy or compact femtosecond laser oscillators. Analytical criteria for designing q preserving cavities are derived. An analysis of a specific MPC consisting of a flat and a curved mirror showed that there is an optimum spot pattern, characterized by the optimum value n_{opt} of n, for which the pulse repetition frequency is minimum. The generalized analysis presented here can be readily applied to MPCs with more complicated mirror geometries. The closed form solutions presented here should enable a more efficient design and optimization of high pulse energy and compact femtosecond lasers.

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