IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Propagation of short pulses through a Bose-Einstein condensate

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2006 J. Phys.: Conf. Ser. 36 194

(http://iopscience.iop.org/1742-6596/36/1/029)

View the table of contents for this issue, or go to the journal homepage for more

Download details:

IP Address: 212.175.32.138
This content was downloaded on 23/06/2017 at 12:10

Please note that terms and conditions apply.

You may also be interested in:

Elastic scattering of a Bose-Einstein condensate at a potential landscape
Iva Bezinová, Joachim Burgdörfer, Axel U J Lode et al.

Gross-Pitaevski equation and resonances in Bose-Einstein condensates
J. da Providencia, A. R. Sakhel, F. B. Malik et al.

Particle creation in bose-einstein condensates: Numerical analysis of the Bogoliubov–de Gennes equation for trapped ultracold atoms
M Kobayashi, Y Kurita, T Morinari et al.

A Method for Filtering and Controlling Soliton States of Bose-Einstein Condensates
R Fedele, P K Shukla, S De Nicola et al.

Anti-continuum approach on nonlinear localized modes in dipolar Bose-Einstein condensates in optical lattice
M H Badarudin, M S A Hadi and B A Umarov

On the theory of light scattering by a dilute gas Bose-Einstein condensate
Yu A Avetisyan and E D Trifonov

Generation of mesoscopic superpositions of a binary Bose-Einstein condensate in a slightly asymmetric double well
C Weiss and N Teichmann

On the theory of superradiant scattering from Bose-Einstein condensate of dilute gas
Yu A Avetisyan and E D Trifonov

Institute of Physics Publishing Journal of Physics: Conference Series **36** (2006) 194–199
doi:10.1088/1742-6596/36/1/029 The 12th Central European Workshop on Quantum Optics

Propagation of short pulses through a Bose-Einstein condensate

Devrim Tarhan[1,2,3] , Seçkin Şefi[1] and Özgür E. Müstecaplıoğlu[2]

[1]Department of Physics, Faculty of Sciences and Letters,
Istanbul Technical University, Maslak 34469, Istanbul, Turkey
[2]Department of Physics, Koç University Rumelifeneri yolu, Sarıyer, Istanbul, 34450, Turkey
[3]Department of Physics, Harran University , Osmanbey Yerleşkesi, Şanlıurfa, Turkey

E-mail: dtarhan@ku.edu.tr

Abstract. We study propagation of short laser pulses in a Bose-Einstein condensate taking into account dispersive effects under the conditions for electromagnetically induced transparency. We calculate dispersion coefficients using typical experimental parameters of slow-light schemes in condensates. By numerically propagating the laser pulse, and referring to theoretical estimations, we determine the conditions for which dispersion starts to introduce distortions on the pulse shape.

1. Introduction

Since the first demonstration of ultra-slow light propagation through a Bose-Einstein condensate (BEC)[1], a major application of slow light has been argued to be an optical memory[2]. Information storage capacity of such an optical memory based upon slowing laser pulses down to subluminal speeds in BECs would strongly depend on the temporal width of the pulses. As the temporal width of the pulses gets smaller, condensate could contain more pulses in a given time interval. Unfortunately, one cannot hope for using shortest pulses (such as femtosecond or even attosecond) available to maximize information capacity. A challenging issue to use such a short pulse in condensates is the fact that slow-light scheme in BEC is based upon electromagnetically induced transparency (EIT)[4, 3] which occurs for not so broad optical frequency window. On the other hand, temporally short pulses have large frequency widths. Present experimental set ups use optical pulses of temporal widths in the order of microseconds. We note that EIT conditions can be reasonably well satisfied for pulses of widths $\sim 10^{-7}$ which would enhance the information capacity of the condensate ten times more.

Another challenging issue in using short pulses in BECs is that their susceptibility to dispersion which may distort the pulse shape. We note that even though dispersive effects have been found small experimentally for microsecond pulses, there is no guarantee that role of dispersion would be small when we make the pulse ten times shorter. As the earlier theoretical models ignore the dispersion completely[5, 6], it is necessary to develop a more general theory of slow light propagation taking into account the higher order dispersive properties of the BEC. In the present paper we develop such a theory, generalizing theory of dispersive EIT in a thermal uniform gas[7]. Using our general theory, we have found that dispersive effects are insignificant for microsecond pulses used in slow-light experiments. However, temporal width of microsecond

© 2006 IOP Publishing Ltd

is the critical width below which dispersion starts to distort the pulse shape. We show that at a width of 10^{-7} seconds, dispersion leads to a small temporal broadening of the pulse.

It may be noted here that study of dispersive effects in slow–short-light propagation may lead to alternative applications such as determination of condensate temperature through measurement of broadening, pulse shape engineering, and frequency band narrowing. While we treat the light classically, novel effects have been predicted for slow-light in quantum regime, in particular a new nonlinearity regime for quantum light[8]. Our results may contribute to such studies as well. Furthermore, the theory in the present paper can easily be extended to situations in which BEC can have an enhanced nonlinear optical response of BEC that might be utilized for dispersion compensation.

The paper is organized as follows. In Sect. 2, Electric susceptibility of an interacting BEC under EIT conditions is derived. In Sect. 3, wave equation including the high order dispersive terms for the EIT medium is derived. We present and discuss our numerical results in section 4. Finally, we conclude in section 5.

2. EIT Electric susceptibility for an interacting BEC

EIT is a technique for making cancellation of induced absorption to weak probe field tuned in resonance to an atomic transition by applying a strong resonant electromagnetic field to couple coherently another atomic transition. Under EIT refractively thick gaseous medium becomes transparent to the probe beam. In order to formulate the effect, we consider a three-level atom, where the atomic states are denoted by $|1\rangle$, $|2\rangle$ and $|3\rangle$, Λ-type atom interacting with two lasers. Interaction of electromagnetic field with such an atom, under electric dipole approximation, can be described by a Hamiltonian of the form $H = H_0 + H_1$, where

$$\begin{aligned} H_0 &= \hbar\omega_1|1><1| + \hbar\omega_2|2><2| + \hbar\omega_3|3><3|, \\ H_1 &= -e\,\vec{r}\cdot\vec{E}(t). \end{aligned} \tag{1}$$

Here, H_0 and H_1 are unperturbed and interaction parts of the Hamiltonian, respectively. Probe laser, with angular frequency ω_p, is tuned to transition $|1>$ to $|3>$ while coupling laser, with angular frequency ω_c, is tuned to transition $|2>$ to $|3>$. We have assumed that only $|3> \rightarrow |1>$ and $|3> \rightarrow |2>$ transitions are dipole allowed. The interaction part of the hamiltonian for the (EIT) system can be calculated by evaluating

$$H_1 = -(|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3|)e\,\vec{r}\cdot\vec{E}(t)(|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3|). \tag{2}$$

Denoting dipole matrix elements $\mu_{31} = \mu_{13}^* = e\langle 3|r|1\rangle$, $\mu_{32} = \mu_{23}^* = e\langle 3|r|2\rangle$, $\mu_{12} = \mu_{21}^* = e\langle 1|r|2\rangle$ and introducing slowly varying amplitudes for the electric fields $E_p(t) = (\varepsilon/2)\exp(-i\omega t)$ and $E_c(t) = (\varepsilon/2)\exp(-i\omega_c t)$, Rabi frequency for the coupling laser can be written as $\Omega_c = \mu_{32}\,\varepsilon/\hbar$. Interaction part of the hamiltonian becomes

$$\begin{aligned} H_1 &= -\frac{\hbar}{2}\{\frac{\mu_{31}\,\varepsilon}{\hbar}\exp(-i\omega t)|3\rangle\langle 1| \\ &+ \frac{\mu_{13}\,\varepsilon}{\hbar}\exp(i\omega t)|1\rangle\langle 3| \\ &+ \Omega_c\exp(-i\omega_c t)|3\rangle\langle 2| + \Omega_c\exp(i\omega_c t)|2\rangle\langle 3|\} \end{aligned} \tag{3}$$

Including the decay terms phenomenologically, to take into account the finite life time of the atomic levels, to the Liouville Equation $i\hbar\dot{\rho} = [H, \rho]$, density matrix equation become[9]

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] - \frac{1}{2}\{\Gamma, \rho\} \tag{4}$$

where Γ is the relaxation matrix,which is defined by the equation $< n \,|\, \Gamma \,|\, m >= \gamma_n \, \delta_{nm}$. At the same time the relation between Γ and ρ is given by $\{\Gamma, \rho\} = \Gamma\rho + \rho\Gamma$. In terms of the matrix elements, explicit equation is given by (4) [9]

$$\dot{\rho_{\rm ij}} = -\frac{i}{\hbar}\sum_{\rm k}(H_{\rm ik}\rho_{\rm kj} - \rho_{\rm ik}H_{\rm kj}) - \frac{1}{2}\sum_{\rm k}(\Gamma_{\rm ik}\rho_{\rm kj} + \rho_{\rm ik}\Gamma_{\rm kj}). \tag{5}$$

The equation of motion for the density matrix elements ρ_{31} and ρ_{21} are given by

$$\begin{aligned}
\dot{\rho_{31}} &= -(i\omega_{31} + \frac{\Gamma_3}{2})\rho_{31} \\
&- \frac{i}{2}\frac{\mu_{31}\varepsilon}{\hbar}\exp(-i\omega t)(\rho_{33} - \rho_{11}) + \frac{i}{2}\Omega_c \exp(-i\omega_c t)\rho_{21} \qquad (6)\\
\dot{\rho_{21}} &= -(i\omega_{21} + \frac{\Gamma_2}{2})\rho_{21} \\
&- \frac{i}{2}\frac{\mu_{31}\varepsilon}{\hbar}\exp(-i\omega t)\rho_{23} + \frac{i}{2}\Omega_c \exp(i\omega_c t)\rho_{31} \qquad (7)
\end{aligned}$$

The levels $|3> \rightarrow |1>$ are coupled by a probe field of amplitude ε and frequency ω , whose dispersion and absorption we are interested in. The dispersion and absorption can be computed by ρ_{31}. All atoms are initially in the ground level $|1>$, $\rho_{11}^{(0)} = 1, \rho_{22}^{(0)} = \rho_{33}^{(0)} = \rho_{31}^{(0)} = 0$. In order to solve equation (6), we can write the equations in the matrix form [9],$\dot{R} = -MR + A$. where R, M and A are matrix elements. As a result R is given by $R = M^{-1}A$.

We found for the density matrix element ρ_{31},

$$\rho_{31} = \frac{i\mu_{31}\varepsilon\exp(-i\omega t)(i\Delta + \Gamma_2/2)}{2\hbar[(\Gamma_2/2 + i\Delta)(\Gamma_3/2 + i\Delta) + \frac{\Omega_c^2}{4}]} \tag{8}$$

Finally, we determine the electric susceptibility χ of BEC consisting of such three-level atoms in EIT scheme. If we use the relationship between electric field, atomic polarizability and (macroscopic) polarization[9], we find

$$\chi = \frac{\rho|\mu_{31}|^2}{\epsilon_0\hbar}\frac{i(i\Delta + \Gamma_2/2)}{[(\Gamma_2/2 + i\Delta)(\Gamma_3/2 + i\Delta) + \frac{\Omega_c^2}{4}]}, \tag{9}$$

where $\rho = \rho(\vec{r})$ stands for the inhomogeneous concentration of atoms in an interacting BEC. At a given temperature density profile of an interacting BEC is composed of two components[10]. Density of the condensate part is evaluated by using Thomas-Fermi approximation, while the density of the thermal component is evaluated semi-classically in a harmonic trap[11]. The total ground state density is given by

$$\rho(\vec{r}) = \frac{\mu - V(r)}{U_0}\Theta(\mu - V(r))\Theta(T_c - T) + \frac{g_{3/2}(ze^{-\beta V})}{\lambda_T^3}. \tag{10}$$

where $U_0 = 4\pi\hbar^2 a_s/m$, m is the atomic mass and a_s is the s-wave scattering length, $\Theta(.)$ is the Heaviside step function, $g_n(x) = \Sigma_j x^j/j^n$, λ_T is the thermal de Bröglie wavelength, $\beta = \frac{1}{k_B T}$, and T_C is the critical temperature. We assume an external trapping potential in the form $V(\vec{r}) = (1/2)m(\omega_r^2 r^2 + \omega_z^2 z^2)$ with ω_r the radial trap frequency and ω_z the angular frequency in the z direction. μ is the chemical potential.

3. Wave equation in dispersive media

In a dispersive media the relation between displacement current vector and electric field is given by

$$D = \epsilon(\omega)E \tag{11}$$

where $\epsilon(\omega) = \epsilon_0[1 + \chi(\omega)]$. Polarization is related to the electric field through electric susceptibility via $P = \epsilon_0\chi(\omega)E$. Susceptibility $\chi(\omega)$ is the Fourier transformation of the response function (Green function) of the medium $\chi(t)$, $\chi(\omega) = \int_{-\infty}^{\infty}\chi(t)\exp(-i\omega t)dt$. Dispersive media are characterized by frequency dependence of susceptibility. Expanding the susceptibility of the dressed atom to second order about a central frequency ω_0, and with $P(\omega - \omega_0) \equiv \epsilon_0\chi(\omega - \omega_0)E(\omega - \omega_0)$, yields[7].

$$\chi(\omega - \omega_0) = \chi(\omega_0) + \frac{\partial\chi}{\partial\omega}|_{\omega_0}(\omega - \omega_0) + \frac{1}{2}\frac{\partial^2\chi}{\partial^2\omega}|_{\omega_0}(\omega - \omega_0)^2, \tag{12}$$

which leads to the polarization[7]

$$P(t) = \epsilon_0\chi(\omega_0)E(t) - i\epsilon_0\frac{\partial\chi}{\partial\omega}|_{\omega_0}\frac{\partial E}{\partial t} - \frac{\epsilon_0}{2}\frac{\partial^2\chi}{\partial^2\omega}|_{\omega_0}\frac{\partial^2 E}{\partial^2 t}. \tag{13}$$

Maxwell equations in the medium are given by

$$\nabla\cdot\mathbf{B} = 0 \quad \nabla\cdot\mathbf{D} = \rho_{free} \tag{14}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \quad \nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t}. \tag{15}$$

In addition, we have the standard constitutive relations $\mathbf{D} = \epsilon\mathbf{E} + \mathbf{P}$, $\mathbf{B} = \mu_0\mathbf{H}$, $\mathbf{J} = \sigma\mathbf{E}$. In such a medium, propagation of an electromagnetic wave is determined by a wave equation in the form

$$-\vec{\nabla}^2\vec{E} + \mu_0\sigma\frac{\partial\vec{E}}{\partial t} + \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial^2 t} = -\mu_0\frac{\partial^2\vec{P}}{\partial^2 t}. \tag{16}$$

Employing the slowly varying amplitude approximation, which is described by conditions,

$$\frac{\partial\varepsilon}{\partial \mathrm{t}} \ll \omega\varepsilon \quad , \quad \frac{\partial\varepsilon}{\partial \mathrm{z}} \ll \mathrm{k}\varepsilon \tag{17}$$

$$\frac{\partial\mathcal{P}}{\partial \mathrm{t}} \ll \omega\mathcal{P} \quad , \quad \frac{\partial\mathcal{P}}{\partial \mathrm{z}} \ll \mathrm{k}\mathcal{P}. \tag{18}$$

wave equation can be reduced to

$$\frac{\partial E}{\partial z} + \alpha E + \frac{1}{V_G}\frac{\partial E}{\partial t} + i\,b_2\frac{\partial^2 E}{\partial^2 t} = 0. \tag{19}$$

Here, E loss term α, group velocity V_G, and dispersion coefficient b_2 are given by [7]

$$\alpha = -\frac{i\pi}{\lambda}\chi(\omega_0), \quad \frac{1}{V_G} = \frac{1}{c} - \frac{\pi}{\lambda}\frac{\partial\chi}{\partial\omega} \tag{20}$$

$$b_2 = \frac{\pi}{\lambda}\frac{1}{2}\left[\frac{\partial^2\chi}{\partial^2\omega}|_{\omega_0}\right]. \tag{21}$$

4. Numerical results and discussion

Let us first compare the relative strength of the last two terms of the wave equation. The third term describes the group velocity dispersion while the last (fourth) one is for the second order dispersion. We have analytically calculated the second order dispersion coefficient b_2. Substituting the numerical values taken from experiment[1], where $a_s = 2.75\,\mathrm{nm}$, $\omega_r = 2\pi \times 69\,\mathrm{Hz}$,$\omega_z = 2\pi \times 21\,\mathrm{Hz}$, we numerically evaluated it to be in the order of $b_2 \sim 10^{-9}\,\mathrm{s}^2/\mathrm{m}$ about the center of a ^{23}Na condensate with the total number of atoms $N = 8.3 \times 10^6$. For some other set of experimentally accessible parameters we have found $b_2 \sim 10^{-8}\,\mathrm{s}^2/\mathrm{m}$. We have also checked third order dispersion coefficient and have found it $\sim 10^{-15}\,\mathrm{s}^3/\mathrm{m}$, which is insignificant. Taking the ratio of the third term to the fourth term, we find that the condition for dispersion to be significant is $(\tau/V_G) \times 10^9 \ll 1$, where τ denotes the temporal pulse width . Subluminal light speeds achieved using ^{23}Na BECs are in the order of $1 - 100(\mathrm{m/s})$. This yields $\tau \ll 10^{-8}$. This explains clearly laser pulses used in present slow-light experiments in condensates do not suffer from dispersion. They are however not the optimal choice for a higher capacity optical memory. One could use a ten times shorter pulse so that $\tau \sim 10^{-7}$. Such a pulse would be influenced by little dispersion in the condensate and would be an optimal choice to use in optical memories. We present our numerical simulation results in Figs.1-2 for the propagation of pulses of widths 10^{-6}s and 10^{-7}s, respectively. In the figures we use dimensionless scaled position (z) and time variables. Position is scaled by the condensate radius while time is scaled by the pulse width. Diameter of the BEC is $\sim 50\,\mu$m. Effect of second order dispersion is a small broadening in the temporal pulse width as demonstrated in Fig.2, where, behavior of the pulse about the center of the condensate, where the dispersive effects are most influential, is shown.

Detailed investigation of behavior of even shorter pulses require more intensive numerical studies and will be published elsewhere. Wave equation is solved via finite difference Crank-Nicholson space marching scheme. The Crank-Nicholson scheme is less stable but more accurate than the fully implicit method; it takes the average between the implicit and the explicit schemes[12]. We use forward difference scheme for the position in order to calculate the next step and we apply central difference scheme for the time. We discretized the wave equation and a set of linear equations are solved at each step to find E^{i+1} where i represents the position z. At each step we find E^{i+1} but this method is too slow. Accordingly, we use Thomas algorithm [12] which is equivalent than gaussian elimination method for tri-diagonal matrices.

5. Conclusion

Ultra-slow light, which is achieved via EIT in BECs, has an effectively long optical path lengths within the condensates. Therefore dispersive effects, in particular temporal broadening of the pulse, can be seen within the short (micrometer) length of BEC. Normally, one would need much longer distances to be covered with the pulse propagating in other more usual medium such as an optical fiber. Though dispersion has no effect on the slowing down the pulse, slow group velocity makes the dispersion visible in such a short distance.

We examined propagation of short laser pulses in a Bose-Einstein condensate taking into account the dispersive effects under the conditions for electromagnetically induced transparency. Our calculation of the high order dispersion coefficients using typical experimental parameters of slow-light schemes show that dispersive effects start to become influential for pulses whose widths are about 10^{-7}s. Present experiments uses microsecond pulses which are below that dispersion limit.

Acknowledgments

We would like to thank N. Postacıoğlu and A. Sennaroğlu for their useful discussions. This work was supported by İstanbul Technical University Foundation (ITU BAP). O.E.M acknowledges support from TUBA-GEBİP Award.

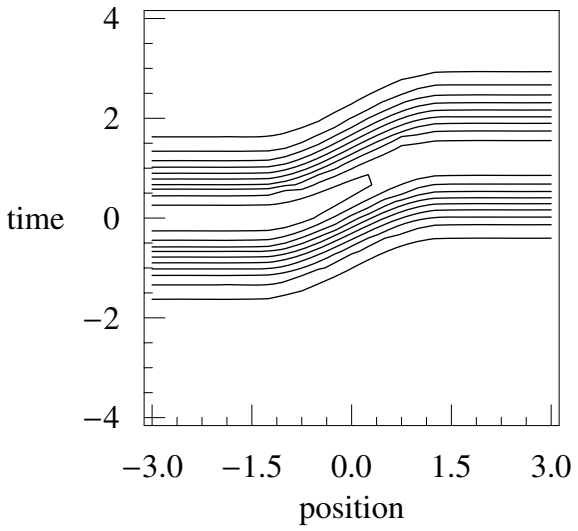


Figure 1. Contour plot of the propagation of an EIT pulse of width $\tau = 10^{-6}$s through a ^{23}Na BEC. Time and position are in dimensionless units as explained in the text.

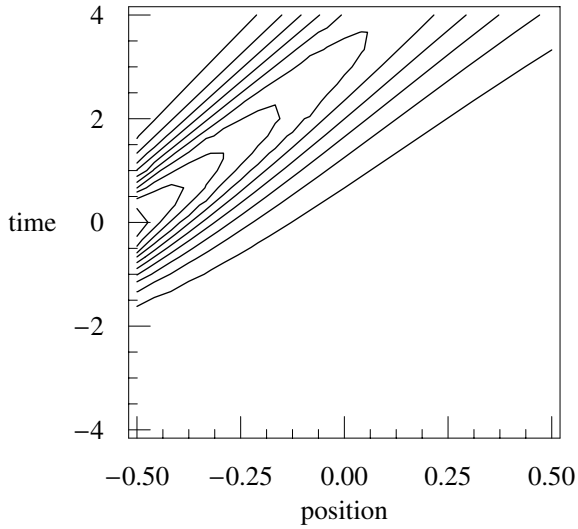


Figure 2. Same with Fig.1 but for $\tau = 10^{-7}$s.

References

[1] Hau L V, Harris S E, Dutton Z and Behroozi C H 1999 *Nature* **397** 594

[2] Fleischhauer M and Lukin M D 2000 Dark-state polaritons in electromagnetically induced transparency *Phys. Rev. Lett.* **84** 5094

[3] Boller K J, Imamoğlu A and Harris S E 1991 Observation of Electromagnetically induced Transparency *Phys. Rev. Lett.* **66** 2593

[4] Harris S E 1997 Electromagnetically induced transparecy *Physics Today* **50** 36-42

[5] Müstecaplıoğlu Ö E and You L 2001 Propagation of raman-matched laser pulses through a Bose-Einstein condensate *Optics Comm.* **193** 301

[6] Müstecaplıoğlu Ö E and You L 2001 Slow light propagation in trapped atomic quatum gases *Phys. Rev. A* **64** 013604

[7] Harris S E, Field J E and Kasapi A 1992 "Dispersive properties of electromagnetically induced transparency *Phys. Rev. A* **46** R29

[8] Lukin M D and Imamoğlu A 2001 Controlling photons using electromagnetically induced transparency *Nature* **413** 273-276

[9] Scully M O and Zubairy M S 1997 *Quantum Optics* (Cambridge: Cambridge)

[10] Naraschewski M and Stamper-Kurn D M 1998 Analytical description of a trapped semi-ideal Bose gas at finite temperatue *Phys. Rev. A* **58** 2423-2426

[11] Bagnato V, Pritchard D E and Kleppner D 1987 Bose-Einstein condensation in an external potential *Phys. Rev. A* **35**, 4354-4358

[12] Garcia A L 2000 *Numerical methods for physics*, (New York: Prentice Hall)

[13] Stauffer D and Aharony A 1994 *Introduction to Percolation Theory*, (London: Taylor and Francis, London) pp.52