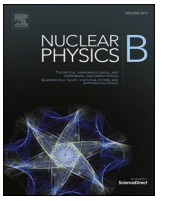
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A quantum universe - antiuniverse emerging from Thirring's conformal nothingness

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ABSTRACT

In this study, the spontaneous emergence of a quantum universe - antiuniverse from zero point (or nothingness) is simulated metaphorically based on Thirring spinor model. To do this, a novel approach was developed to demonstrate how the quantum universe and its anti-universe emerge at the zero-point conformal of the massless Thirring spinor model. The model's phase-space dynamics are characterized by its attractors, and quantified via Boltzmann–Gibbs–Shannon entropy, relative entropy, and the largest Lyapunov exponent. The results were analysed for their cosmological implications with reference to the Coleman–De Luccia instanton, the Hawking–Turok instanton and CPT-symmetric universe models.

1. Introduction

Various types of cosmological instantons have been postulated for the purpose of describing the fundamental initial conditions of realistic universes or the emergence of a quantum universe. Most notably, Stephen Hawking and Ian Moss proposed a definition of an instanton (the Hawking-Moss instanton) [1] to construct a model of the universe in the context of the idea of 'no boundary', which describes a situation in which the universe could come into existence spontaneously from nothing [2]. The Coleman-De Luccia (CDL) instanton [3] is the basis of how an open Universe is formed through bubble nucleation, which represent a false vacuum the matter in the early Universe. In 1998, Stephen Hawking and Neil Turok developed a new type of instanton, which describes the initial conditions of a realistic universe and suggests that the emergence of an infinite open universe should be modelled via a 'pea'-sized instanton [4]. However, the Hawking–Turok (HT) instanton has a singularity which introduces challenges to its physical interpretation.

Recently, a quantum universe model called the 'Thirring Universe Model' has been proposed based on the instantons of quantum field theory to describe the fundamental initial conditions of realistic universes [5]. The proposed quantum universe [5] was defined on a field composed of instantons using the massless Thirring model [6]. The massless Thirring model is a widely recognised exemplar of a quantised toy model that exhibits the characteristics of the quantisation of relativistic quantum field theories and has conformal symmetry [7–11]. The massless Thirring model has stable fermionic instanton solutions of the form $(\mp a, \mp a)$ and $(0,0)$ [12–14]. Here, the zero-instanton point (0,0) represents a degenerate conformal fixed point, where the model remains identically zero for all values of the coupling parameter. By changing the coupling value of the model, quantum waves can be created starting from these instanton points outside the zero instanton [5,15].

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Unlike other cosmological instanton models, this study proposes a symmetric dual quantum universe model that may spontaneously arise from the zero-instanton point of the Thirring model, interpreted as nothingness. In this context, "Thirring's conformal nothingness" is defined as the point at which all values are zero. The zero-instanton point of the Thirring model corresponds to a degenerate conformal fixed point, and the new approach developed to overcome this degenerate behaviour is presented in Section 2.2. The primary objective is to generate quantum waves that, starting from zero origin or from nothingness, lead to the formation of matter and antimatter pairs.

For this purpose, the matter term was added to the massless Thirring model reduced to dimensionless form [14] where the massless Thirring model instanton values were assigned to the added matter terms to start the model from the (0,0) instanton point. Quantum fluctuations were created by perturbing the model, which is in the instanton state at (0,0) point, with the coupling - interaction parameter αAB. This allowed us to plot the behaviour of matter and antimatter particles in phase space, starting from the (0,0) instanton point. To characterise the system's dynamics, we calculated the Boltzmann-Gibbs-Shannon (BGS) entropy, relative entropy and the largest Lyapunov exponent values for the cyclic attractors of the model. Our approach was then evaluated with reference to the CDL instanton [3], the HT instanton [4] and the 'CPT symmetric universe' model [16].

2. Theoretical basis

2.1. The massless Thirring model

The massless Thirring model, which describes a self-interacting Dirac fermion in two dimensions, was first introduced by Walter E. Thirring in 1958, with the use of the Lagrange density [6]

$$\mathscr{L} = \overline{\psi}\left(i\gamma^{\mu}\partial_{\mu}\right)\psi - \frac{g}{2}(\overline{\psi}\gamma^{\mu}\psi)^{2} \tag{1}$$

Where ψ is a spinor field with two components ψ_1 and ψ_2. The γ^{μ} for $\mu = 1;2$ are the Pauli matrices, which correspond to σ_1, σ_2 and σ_3. The $\overline{\psi}$ is the adjoint of ψ. The ∂_{μ} represents the partial derivative. The g is a dimensionless coupling constant that measures how strongly fermions interact, $g > 0$.

The complex Euclidean structure underlying the Heisenberg ansatz of ψ [17] is written as $\psi = \left[iX_{\mu}\sigma_{\mu}X(s) + \varphi(s)\right]c$. Where $X(s)$ and $\varphi(s)$ are real functions of $s = x^2 + t^2$, $x_1 = x$, $x_2 = t$. The c is an arbitrary spinor constant. The following nonlinear differential equation system [14] is obtained from the equation of motion (1) for $\mathscr{L} = 0$ [17,18].

$$2X + 2s\frac{dX}{ds} - \alpha\left[sX^2 + \varphi^2\right]\varphi = 0 \tag{2a}$$

$$2\frac{d\varphi}{ds} + \alpha\left[sX^2 + \varphi^2\right]X = 0 \tag{2b}$$

Using the transformations in Ref. [18]; $X = As^{-\sigma}F(Z)$, $\varphi = Bs^{-\tau}G(Z)$ and $Z = lns$, with $\sigma = \tau + \frac{1}{2}$, $\tau = \frac{1}{4}$ and $A^2 = B^2$. The dimensionless form of the nonlinear differential equation systems (2) are achieved as follows [14]:

$$2\frac{dF}{dZ} + \frac{1}{2}F - \alpha AB\left[F^2 + G^2\right]G = 0 \tag{3a}$$

$$2\frac{dG}{dZ} - \frac{1}{2}G + \alpha AB\left[F^2 + G^2\right]F = 0 \tag{3b}$$

Where the fields $F(Z)$ and $G(Z)$ are dimensionless, and the coupling $\alpha AB > 0$. The fixed points of the dimensionless fields (3) are found as $(0.5,\ 0.5)$, $(0,0)$ and $(-0.5,\ -0.5)$ when $\alpha AB = 1$ [14]. These fixed points embody the stable fermionic instanton solutions as formulated by Akdeniz and Smailagiç. If generalised, the equations (3) have instanton solutions at $(+a, +a)$, $(0,0)$ and $(-a,\ -a)$ for all other values of αAB, $a \in \mathbb{R}$ [12,13]. Here, the point (0,0) is a degenerate fixed point, where the model remains identically zero, independent of the coupling parameter αAB.

To set the frequency of the Thirring oscillator, the frequency transformation was carried out within the equations system (3). Let $Z = wz$, w denoting the angular frequency, $F_* = F(wz)$ and $G_* = G(wz)$

$$\frac{dF_*}{d(wz)} = \frac{1}{w}\frac{dF_*}{dz} \tag{4a}$$

$$\frac{dG_*}{d(wz)} = \frac{1}{w}\frac{dG_*}{dz} \tag{4b}$$

And,

$$\frac{2}{w}\frac{dF_*}{dz} + \frac{1}{2}F_* - \alpha AB\left[{F_*}^2 + {G_*}^2\right]G_* = 0 \tag{5a}$$

$$\frac{2}{w}\frac{dG_*}{dz} - \frac{1}{2}G_* + \alpha AB\left[F_*{}^2 + G_*{}^2\right]F_* = 0 \tag{5b}$$

Then;

$$\frac{dF}{dz} = w\left(-\frac{1}{4}F + \frac{\alpha AB}{2}\left[F^2 + G^2\right]G\right) \tag{6a}$$

$$\frac{dG}{dz} = w\left(\frac{1}{4}G - \frac{\alpha AB}{2}\left[F^2 + G^2\right]F\right) \tag{6b}$$

Fig. 1 shows the massless Thirring model attractor plotted in the range of $1 \leq \alpha AB \leq 3$ for the equation (6), $w = 1$. This system of Eqs. (6a-6b) was solved numerically using instanton solutions at $\alpha AB = 1$ as its initial conditions. The attractor of the system (6) has two fixed points of $(0.5,\ 0.5)$ and $(-0.5, -0.5)$ for $\alpha AB = 1$. The system (6) has cyclic attractors for $\alpha AB > 1$ states.

2.2. Adding matter to the massless Thirring model

The massless Thirring model (6) will be revised to include additional effective terms representing matter–antimatter excitations, in order to remove the degeneracy of the fixed point at (0,0). The model can be defined as follows:

$$F'(z) \equiv F(z) + dm1 \text{ and } G'(z) \equiv G(z) + dm2,\ (dm1 \text{ and } dm2 \in \mathbb{R}). \tag{7}$$

They (7) can be written as $F(z) \equiv F'(z) - dm1$ and $G(z) \equiv G'(z) - dm2$. When these expressions are substituted into Eq. 6a-6b, the following system of Eqs. (8a-8b) is obtained:

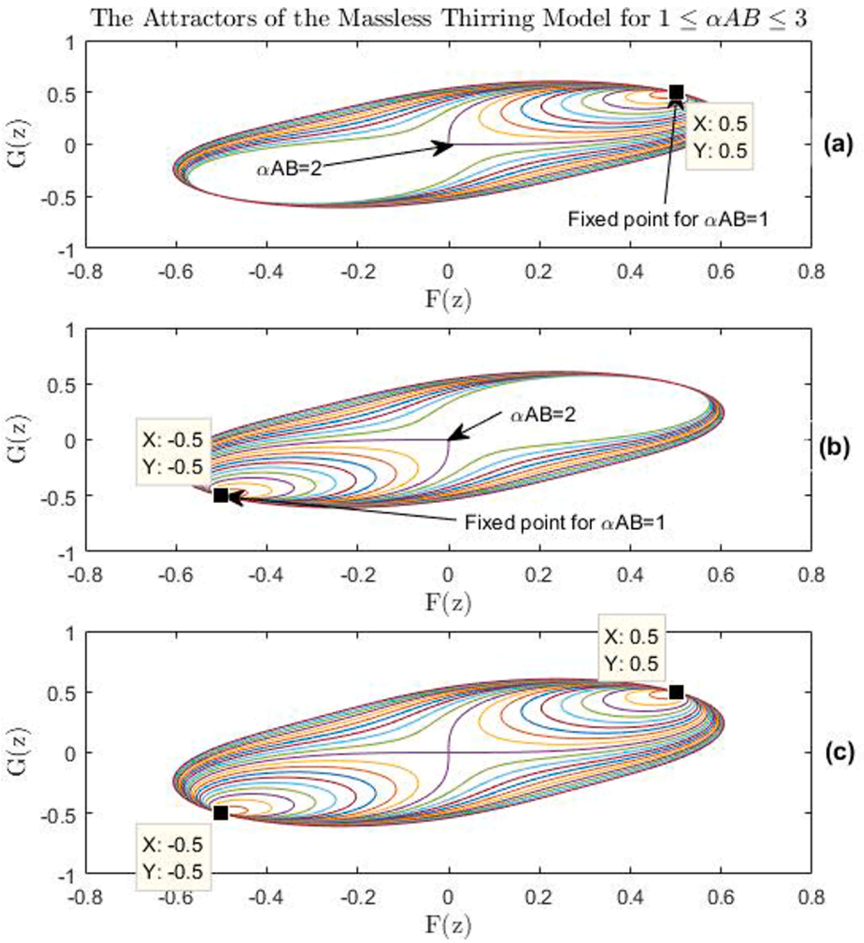


Fig. 1. The massless Thirring model attractor plotted in the range of $1 \leq \alpha AB \leq 3$ (a) The massless Thirring attractor plotted with fixed points $F(0) = 0.5$ and $G(0) = 0.5$. (b) The massless Thirring attractor plotted with fixed points $F(0) = -0.5$ and $G(0) = -0.5$. (c) The massless Thirring attractors in (a) and (b) are shown from the same phase space.

$$\frac{dF}{dz} = w\left(-\frac{1}{4}(F' - dm1) + \frac{\alpha AB}{2}\left[(F' - dm1)^2 + (G' - dm2)^2\right](G' - dm2)\right) \tag{8a}$$

$$\frac{dG'}{dz} = w\left(\frac{1}{4}(G' - dm2) - \frac{\alpha AB}{2}\left[(F' - dm1)^2 + (G' - dm2)^2\right](F' - dm1)\right) \tag{8b}$$

The massless Thirring spinor model in equation (6) admits three instanton solutions, located at $(+a, +a)$, $(0,0)$ and $(-a, -a)$, for all values of the coupling parameter $\alpha AB > 0$, $a \in \mathbb{R}$. Among them, the origin (0,0) corresponds to a degenerate fixed point of the effective action, for which the solution remains identically zero for all values of αAB. It is generally expected that when a system is transported from one stable fixed point to another, it regains stability after settling into the new configuration. Based on this general assumption, the Thirring model can be driven to the origin (0,0) by assigning the other fixed-point values to the variables *dm1* and *dm2* in the system of equations (8), namely $dm1 = dm2 = \pm a$. If the model (8) is subsequently perturbed by the coupling parameter αAB at the origin (0,0), it develops oscillatory behaviour. As a result, the behaviour of matter and antimatter components originating from the origin (0,0) can be analysed in the phase-space representation. In this sense, the reduced dynamical formulation provides a regularized description of the massless Thirring model (6), effectively resolving the degeneracy. In this sense, this approach provides a regularized description of the massless Thirring model (6) by removing the degeneracy of the stationary fixed point at the origin.

3. Methods

3.1. Quantum entropy

The concept of quantum entropy was first introduced by Johann von Neumann in 1932. The quantum entropy is a quantitative measure of the degree of statistical unpredictability inherent in the description of a quantum system. This entropy is based on a density matrix (or density operator), called ρ, which encompasses the probability element inherent in a system within the quantum mechanics domain. For a quantum-mechanical system described by a density operator ρ, the quantum entropy is [19–21]:

$$H(\rho) = -Tr(\rho \ln \rho) \tag{9}$$

In this context, ρ stands for the one-particle reduced density matrix, and Tr denotes the trace operation over that matrix. Following the process of spectral diagonalisation of ρ

$$\rho = \sum_i p_i |\psi^i\rangle\langle\psi^i| \tag{10}$$

Where $\{|\psi^i\rangle\}$ is a set of orthonormal eigenvectors and $p_i > 0$. The Eq. (9) can be written in the following matrix form [19];

$$\rho \ln \rho = \begin{pmatrix} p_1 \ln p_1 & & & \\ & p_2 \ln p_2 & & \\ & & p_3 \ln p_3 & \\ & & & \ddots \end{pmatrix} \tag{11}$$

This matrix can likewise be written in the standard Boltzmann-Gibbs-Shannon (BGS) entropy form for the probability distribution $\{p_i\}$, which satisfies $H(\rho) \geq 0$.

$$H(\rho) = -\sum_i p_i \ln p_i \tag{12}$$

3.2. Quantum relative entropy

The quantum relative entropy [22–24] is utilised to quantify the statistical distinguishability of two quantum states. Let the density matrices be designated as ρ and σ. The quantum relative entropy of ρ with regard to σ is hereby defined as follows:

$$H(\rho \setminus \sigma) = Tr(\rho \ln \rho) - Tr(\rho \ln \sigma) = Tr\left(\rho \ln \frac{\rho}{\sigma}\right) \tag{13}$$

Here, $H(\rho \setminus \sigma) \geq 0$. if $\rho = \sigma$, $H(\rho \setminus \sigma) = 0$

3.3. Calculating entropy of the Thirring model

The BGS entropy of the Thirring model was computed using the technique of scalogram entropy [25,26]. Initially, the energies of the Thirring oscillations were obtained from the scalogram S(s), denoting the energy of the signal *f* as expressed through its continuous wavelet transform (CWT) [27]. Subsequently, the BGS entropy was calculated from the probability distribution of the normalized inner

scalogram $\overline{S}^{inner}(s)$.

$$S(s)=\|\langle f,\psi_{u,s}\rangle\|=\|\ Wf(u,s)\|=\left(\int_{-\infty}^{+\infty}|Wf(u,s)|^2du\right)^{1/2} \tag{14}$$

Where, the CWT of a signal f is defined as $Wf(u,s)$, The time translating u and scale dilating s of a wavelet function are denoted by the symbol $\psi_{u,s}$.If the scalogram in Eq. (14) is expressed for a finite time interval $I=[a,b]$;

$$S^{inner}(s)=\|\ Wf(u,s)\|_{J(s)}=\left(\int_{c(s)}^{d(s)}|Wf(u,s)|^2du\right)^{1/2} \tag{15}$$

Here $J(s)=[c(s),d(s)]\subseteq I$ is the maximal subrange in I. The inner scalogram $S^{inner}(s)$ gives information regarding the alterations in energy densities at varying scales of the signal over time. The averaged distribution of energy density of the signal is derived as independent of time by normalizing the inner scalogram. This approach enables the comparison of average energy densities across different scales s. The normalized inner scalogram $\overline{S}^{inner}(s)$ can be expressed as follows [28]

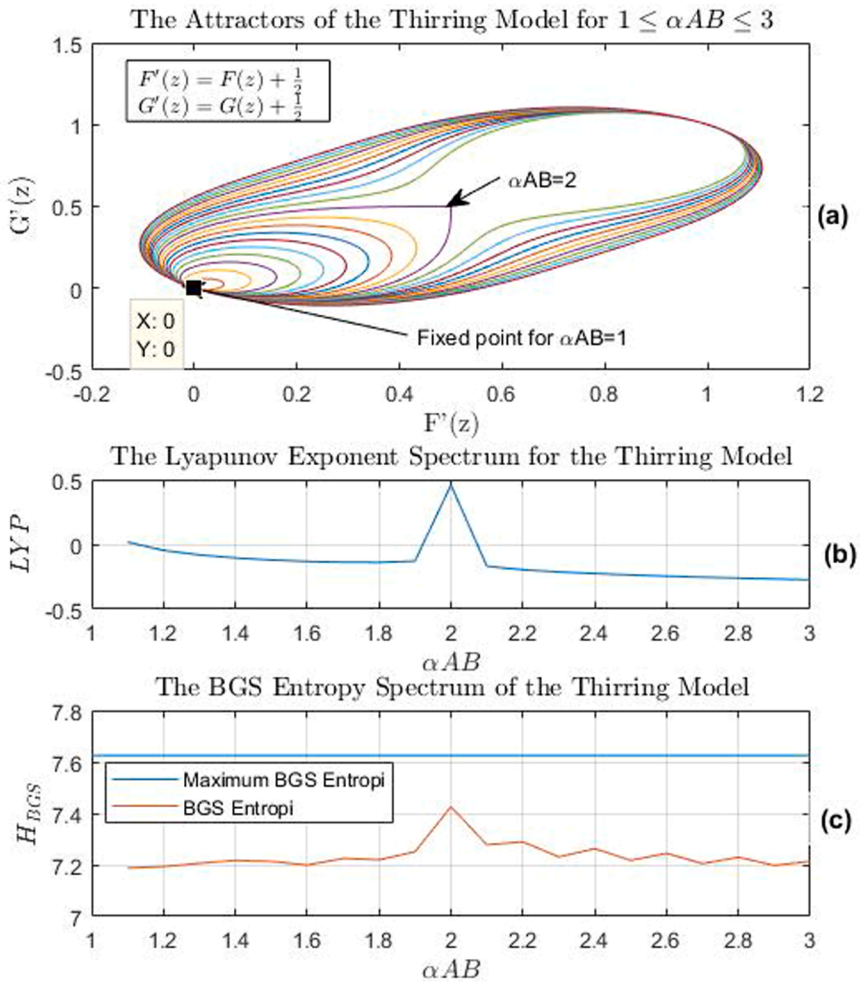


Fig. 2. (a) The attractors of the Thirring model (8) plotted for $F'(z)=F(z)+\frac{1}{2}$ and $G'(z)=G(z)+\frac{1}{2}$. The initial conditions were chosen as $F'(0)=0$ and $G'(0)=0$. The attractor of the spinor model is a conformal fixed point, defined by initial conditions (0,0) for $\alpha AB=1$. The model has the cyclic attractors for the range of $1<\alpha AB\leq 3$. (b) The largest Lyapunov spectrum calculated from the time series of $F'(z)$ for the range of $1<\alpha AB\leq 3$. (c) The BGS entropy spectrum calculated from the time series of $F'(z)$ for the range of $1\leq\alpha AB\leq 3$. The BGS entropy value is zero at $\alpha AB=1$.

$$\overline{S}^{inner}(s) = \frac{S^{inner}(s)}{(d(s) - c(s))^{\frac{1}{2}}} = (\overline{E_1}, \overline{E_2}, \overline{E_3}, \ldots, \overline{E_s}) \tag{16}$$

Where $\overline{E_s}$ is defined as the average energy of the signal for each scale s. The normalized inner scalogram (16) expresses the mean energy distributions of the signal at varying levels of scale, whilst excluding temporal information. The probability distribution (17) can be written in the form of the Boltzmann distribution;

$$p_s = \frac{e^{-\beta \overline{E}_s}}{\sum_s e^{-\beta \overline{E}_s}}; \sum_s p_s = 1 \tag{17}$$

Where p_s denotes the probability of energy at each scale s, The value of β is a positive. The BGS entropy (12) can be calculated through the probability distribution given in Eq. (17). It can also be rewritten as;

$$H'_{BGS} = -k \sum_s p_s \ln p_s \tag{18}$$

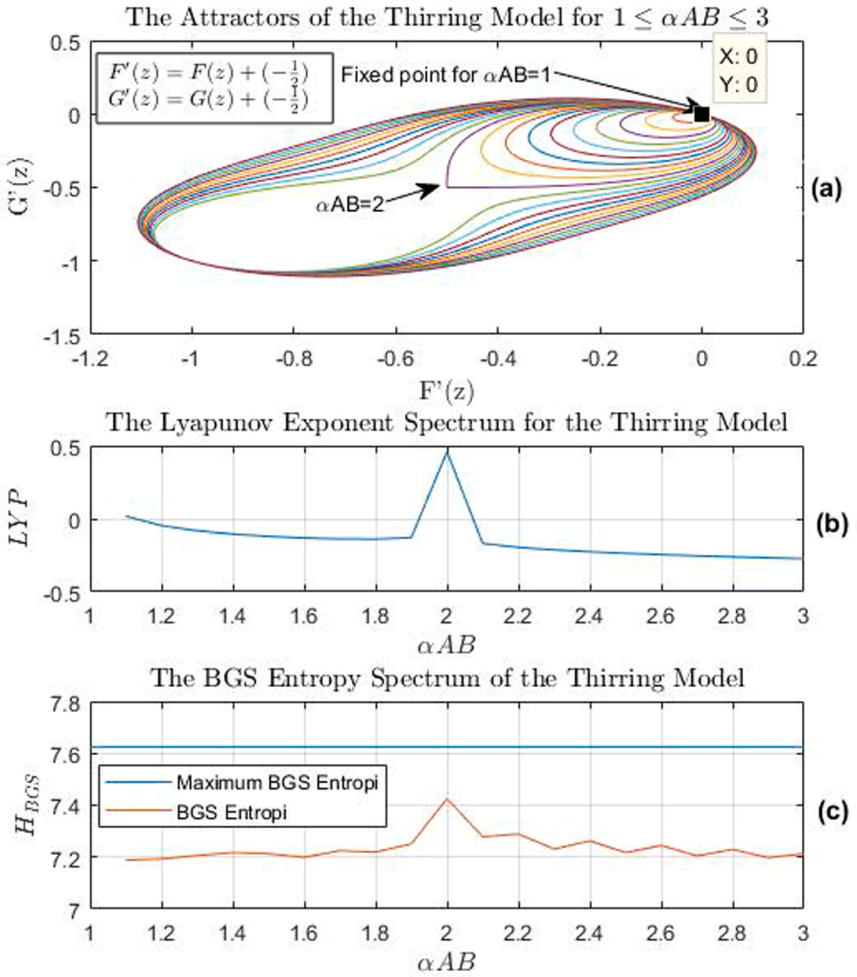


Fig. 3. (a) The attractors of the Thirring model (8) plotted for $F'(z) = F(z) + \left(-\frac{1}{2}\right)$ and $G'(z) = G(z) + \left(-\frac{1}{2}\right)$. The initial conditions were chosen as $F'(0) = 0$ and $G'(0) = 0$. The attractor of the spinor model is a conformal fixed point, defined by initial conditions (0,0) for $\alpha AB = 1$. The model has the cyclic attractors for the range of $1 < \alpha AB \leq 3$. (b) The largest Lyapunov spectrum calculated from the time series of $F'(z)$ for the range of $1 < \alpha AB \leq 3$. (c) The BGS entropy spectrum calculated from the time series of $F'(z)$ for the range of $1 \leq \alpha AB \leq 3$. The BGS entropy value is zero at $\alpha AB = 1$.

3.4. Lyapunov exponent

The Lyapunov exponent λ quantifies the predictability of the of a dynamical system's behaviour. It is the average of the natural logarithmic rate of divergence or convergence of the orbits of nearby orbits in phase space [29,30]. The Lyapunov exponents of the Thirring oscillations $F'(z)$ are calculated from the derivatives $\frac{dF'(z)}{dz}$ as follows

$$\lambda = \lim_{n\to\infty}\frac{1}{n}\sum_{i=0}^{n-1}\ln\left|\frac{dF'(z)}{dz}\right| \tag{19}$$

If $\lambda < 0$, the system is periodic, meaning that the behavior of the system is predictable. However, If $\lambda > 0$, the system oscillates chaotically, so the behavior of the system is unpredictable. This means that the system is sensitive to a very small perturbation of the initial state [31].

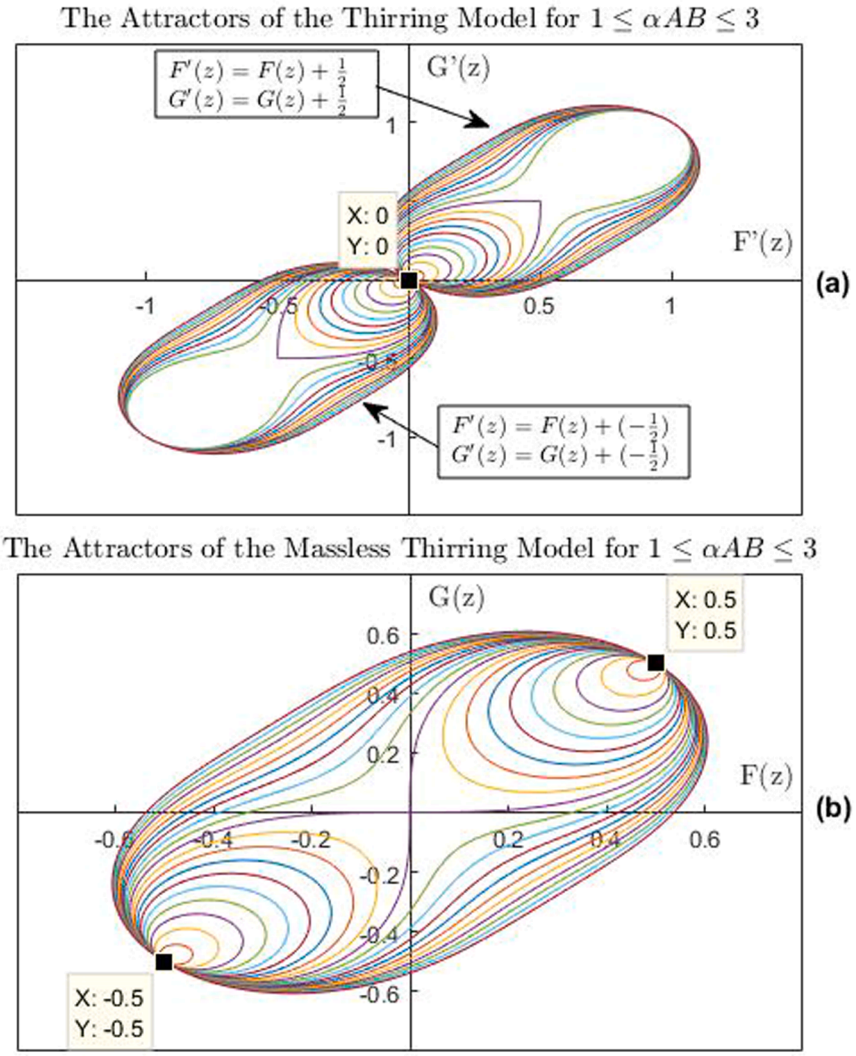


Fig. 4. Comparison of the attractors of the massless Thirring spinor model (6) and the extended model with added matter (8). (a) The attractors of the extended Thirring model (8) plotted by starting at the fixed point that $F'(0) = 0$ and $G'(0) = 0$, for $F'(z) = F(z) \mp \frac{1}{2}$ and $G'(z) = G(z) \mp \frac{1}{2}$. The attractor of the spinor model is a conformal fixed point, defined by initial conditions (0,0) for $\alpha AB = 1$. The model has the cyclic attractors for the range of $1 < \alpha AB \leq 3$. (b) The attractors of the massless Thirring model (6) plotted by starting at two different fixed points that $F(0) = \mp\frac{1}{2}$ and $G(0) = \mp\frac{1}{2}$. The attractor of the spinor model consists of two fixed points serving as initial conditions, located at (0.5, 0.5) and (-0.5, -0.5) for the parameter $\alpha AB = 1$. The model exhibit cyclic attractors for the range $1 < \alpha AB \leq 3$.

4. Results

The system of nonlinear differential equations (8) represents the massless Thirring model extended by the inclusion of matter terms. As demonstrated in the massless Thirring model (6) [6,12–14], the model admits fixed-point configurations located at (0.5, 0.5), (0,0) and (− 0.5, − 0.5) for the coupling value set to $\alpha AB = 1$.The added matter values were chosen as the other two fixed-point values to initialize the Thirring model (8) at the origin (0,0). Accordingly, the fields were shifted as $F'(z) = F(z) \mp \frac{1}{2}$ and $G'(z) = G(z) \mp \frac{1}{2}$ in order to transport the fixed points to the origin (0,0). In this particular instance, the model (8) remains stable, as stability is preserved when a system is transported from one fixed point to another. The system of nonlinear differential equations (8) was numerically solved via the fourth-order Runge–Kutta algorithm [32]. In equation (8), the initial conditions were specified as $F'(0) = 0$ and $G'(0) = 0$, $w = 1$.

Fig. 2(a) shows the phase-space attractors corresponding to the Thirring model (8), plotted for $F'(z) = F(z) + \frac{1}{2}$ and $G'(z) = G(z) + \frac{1}{2}$. The model (8) maintains stable state at (0,0) for $\alpha AB = 1$. Here, the attractor of the model is a fixed point, defined as (0,0). The model (8) will start to oscillate when it is perturbed at the origin (0,0) for values of $\alpha AB > 1$. As demonstrated in Fig. 2(a), the cyclic attractors corresponding to the model (8) were examined within the parameter range of $1 < \alpha AB \leq 3$ with steps of 0.1.

Fig. 3(a) shows the Thirring model (8) attractors plotted for $F'(z) = F(z) + \left(-\frac{1}{2}\right)$ and $G'(z) = G(z) + \left(-\frac{1}{2}\right)$. The model (8) maintains stable state at (0,0) for $\alpha AB = 1$. Here, the attractor of the model is a fixed point, defined as (0,0). The model (8) will start to oscillate when it is perturbed at the origin (0,0) for values of $\alpha AB > 1$. As demonstrated in Fig. 3(a), the cyclic attractors corresponding to the model (8) were examined within the parameter range of $1 < \alpha AB \leq 3$ with steps of 0.1.

The Thirring model (8) attractors plotted in Figs. 2(a) and 3(a) are shown from the same phase space in Fig. 4(a). The attractors appear to be mirror images or symmetrical with respect to each other.

The Figs. 2(b) and 3(b) indicate the values of the largest Lyapunov exponent calculated from the time series of $F'(z)$ that were obtained in the range of $1 < \alpha AB \leq 3$. The calculation of Lyapunov exponent value is not possible due to the presence of a point at $\alpha AB = 1$. The largest Lyapunov exponent peaks at $\alpha AB = 2$.

The Figs. 2(c) and 3(c) show the BGS entropy calculated from the probability distribution of the normalized inner scalogram, derived from the time series of $F'(z)$ that were obtained in the range of $1 \leq \alpha AB \leq 3$. The Haar wavelet was employed, using integer scales ranging from $s_0 = 1$ to $s_1 = 2048$. The BGS entropy value is zero at αAB=1 and reaches its highest value at αAB=2.

Table 1 indicates the relative entropy values calculated from the time series of $F'(z)$ that were obtained for some values of αAB.

5. Discussion and conclusion

This study considers the Thirring spinor model as a metaphor for the universe. It proposes that the quantum universe, along with its anti-universe, may have spontaneously emerged from the zero point (or nothingness). To this end, a novel approach was developed to describe the formation of matter and antimatter particles at the zero point conformal of fermionic instantons using the massless Thirring model. In this context, the zero instanton point of the Thirring model, where all values are zero, was defined as the Thirring's conformal nothingness.

The massless Thirring model (6), which is reduced to the dimensionless form of the nonlinear differential equation, has fixed point solutions located at $(+a, +a)$, $(0,0)$ and $(-a, -a)$ for values of $\alpha AB > 0$, which represents a dimensionless coupling constant. If the model in stable state is perturbed by αAB (or its value is changed), the model starts to generate quantum fluctuations at the fixed point that $(\mp a, \mp a)$, as shown Fig. 1. However, since the model is in a continuous fixed point state at the origin (0,0), it cannot be perturbed by the αAB coupling parameter. At this point, the model remains identically zero for all values of αAB, reflecting the presence of a degeneracy at the origin. The degenerate fixed point of the model at (0,0) was removed using the following approach.

The massless Thirring model was revised by the addition of a matter term, $F'(z) \equiv F(z) + dm1$ and $G'(z) \equiv G(z) + dm2$. A dynamical system transported from one stable state to another remains stable. If the fixed point values are given to the matter values added to the motion equation of the massless Thirring model, $dm1 = dm2 = \mp a$, the model's fixed point configurations are moved to the degenerate fixed point (0,0). This allows us to perturb the model with the parameter αAB at the origin (0,0). The model creates quantum fluctuations starting at (0,0) in response to a change in the αAB value. Thus, the formation of matter and antimatter particles at the origin (0,0) and the behavioural differences in phase space might be observed. In the extended model (8), since all matter and antimatter particles share an instanton state at the point (0,0), it is natural to interpret this point as the instanton state of the universe itself, referred to as Thirring conformal nothingness.

Fig. 4(a) shows the attractors of the massless Thirring model with added matter (8) plotted for $F'(z) = F(z) \mp \frac{1}{2}$ and $G'(z) = G(z) \mp \frac{1}{2}$. The model has a point attractor defined as the (0,0) fixed point for $\alpha AB = 1$. The model exhibits cyclic attractors starting at (0,0) in response to alterations in the range of $1 < \alpha AB \leq 3$. It is clearly seen from Fig. 4(a) that the attractors obtained for matter and antimatter are mirror images of each other with respect to the origin (0,0). This outcome is consistent with the expected behaviours of

Table 1
Relative entropy values calculated for some states of αAB.

Relative Entropy: $H(\alpha AB = x \setminus \alpha AB = y)$						
$H(1.6 \setminus 1.7)$	$H(1.7 \setminus 1.8)$	$H(1.8 \setminus 1.9)$	$H(1.9 \setminus 2.0)$	$H(2.0 \setminus 2.1)$	$H(2.1 \setminus 2.2)$	$H(2.2 \setminus 2.3)$
0.0	0.0	0.0025	0.1061	0.0462	0.0046	0.0033

matter and antimatter particles.

As illustrated in Figs. 2(a) and 3(a), the cyclic attractor corresponding to the model for αAB=2 is a homoclinic orbit. The BGS entropy and the largest Lyapunov exponent values calculated for the cyclic attractors corresponding to the model reach their peaks in the homoclinic orbit, as demonstrated in Figs. 2(b)-(c) and 3(b)-(c). This phenomenon is further evidenced by the relative entropy value presented in Table 1. It is observed that the model transitions to a large-scale periodic phase state with an increase in entropy. This suggests the hypothesis that fermionic-type particles might be formed due to the increasing entropy of quantum fluctuation interactions [15].

The Thirring universe model [5] is based on a field composed of fixed-point configurations of the massless Thirring model (6). As illustrated in Fig. 4(b), this field consists of fixed-point pairs that are symmetrically distributed about the origin (0,0), which corresponds to the degenerate fixed point of the model. This study demonstrates that, once the degenerate fixed point of the Thirring model is eliminated, quantum waves that lead to all matter and antimatter particles can originate at the origin (0,0), as shown in Fig. 4(a). Therefore, the present study puts forward the hypothesis that the quantum universe and its anti-universe, which are mirror images of each other, emerge from the Thirring's conformal nothingness. In other words, this quantum universe model exhibits an intrinsic duality, emerging from nothingness. The creation of the quantum universe in the Thirring's conformal nothingness can be characterised by the concept of "zuhur" [33,34] reflecting its association with "unpredictable emergence". Furthermore, Fig. 4(a) provides a possible explanation to the question, 'Why is there so little antimatter in the present universe?' [35,36].

Let's examine the findings of this study with reference to the HT instanton [4,37,38], which describes the initial conditions of a realistic universe and suggests that the emergence of an infinite open universe should be modelled via a 'pea' sized instanton with a singularity. The concept posits that the inflation of space-time is associated with quantum jumps uphill in the instanton. As shown in Figs. 2(a), 3(a) and 4(a), in the event that origin (0,0) of the Thiring model is regarded as the singularity of the HT instanton, it can be assumed that the cyclic attractor in the interval of $1 \leq \alpha AB \leq 2$ may correspond to the pea-sized HT instanton. It can also be posited that the increase in entropy occurring at $\alpha AB = 2$ may correspond to the quantum jumps uphill expressed in the HT instanton. However, the quantum fluctuations in the HT instanton end in a singularity. This study shows that if the Thirring model at its conformal fixed point is perturbed via the coupling parameter αAB, quantum fluctuations starting from the origin (0,0) can be generated.

The CDL instanton [3] is a theoretical solution in cosmology and quantum field theory. It describes how a false vacuum can decay into a true vacuum in a gravitational context, typically in the early universe. It predicts multiverse models, i.e., that each nucleated bubble can become a separate universe. However, this study predicts the existence of merely two possible cases in which the universe and its anti-universe – like mirror images of each other.

As can be seen in Fig. 4(a), it is clearly seen that the attractors exhibiting the behaviour of matter and antimatter are symmetric or mirror images of each other at the origin (0,0). These findings support the predictions of the 'CPT symmetric universe' model [16], which posits that the pre- and post-explosion periods consist of a universe/anti-universe pair that emerges from nothingness into a hot, radiation-dominated state.

CRediT authorship contribution statement

Mahmut Akilli: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Conceptualization. **Nazmi Yılmaz:** Writing – review & editing. **K. Gediz Akdeniz:** Writing – original draft, Supervision, Methodology, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare no competing interests.

Data availability

Data will be made available on request.

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