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PRICING AND LOT-SIZING DECISIONS FOR PERISHABLE PRODUCTS WHEN DEMAND CHANGES BY FRESHNESS

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Abstract. Perishable products like dairy products, vegetables, fruits, pharmaceuticals, etc. lose their freshness over time and become completely obsolete after a certain period. Customers generally prefer the fresh products over aged ones, leading the perishable products to have a decreasing demand function with respect to their age. We analyze the inventory management and pricing decisions for these products, considering an age-and-price-dependent stochastic demand function. A stochastic dynamic programming model is developed in order to decide when and how much inventory to order and how to price these products considering their freshness over time. We prove the characteristics of the optimal solution of the developed model and extract managerial insights regarding the optimal inventory and pricing strategies. The numerical studies show that dynamic pricing can lead to significant savings over static pricing under certain parameter settings. In addition, longer replenishment cycles are seen under dynamic pricing compared to static pricing, even though similar quantities are ordered in each replenishment.

1. **Introduction.** Management of perishable products like dairy products, vegetables, fruits, pharmaceuticals, etc. have been a very interesting topic since the first research in the 1950s. Since these products deteriorate over time, their demand changes over their lifetime and they become obsolete after a certain amount of time, and this makes the inventory management of these products much more complicated. Billions of dollars' worth of products become obsolete each month due to being unsold at the end of their expired life as mentioned by Minner and Transchel [37] and Hengyu et al. [25]. In today's world, demand structure for perishable products is less predictable, and more uncertainty leads to less profitability.

As stated by Wang and Li [48], due to their nature, the quality of perishable products can be considered as a dynamic state that decreases continuously until the point when it is unfit for sale or consumption. Aging of products decreases the demand rate due to the changing customer preferences for freshness. In this study, we focus on coordinated dynamic pricing and inventory replenishment decisions for perishable products considering a continuous review inventory model, and develop a

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dynamic programming model for this problem in order to find the optimal solution for the replenishment time, order amount, and pricing decisions.

When there is a certain amount of inventory in stock at a certain age, the managers need to decide whether to order a new fresh batch or to continue with the current stock for some more time. If they continue with the current aged stock, the demand for these products might be low at the current price and sales might be low or they need to decrease the price in order to increase the demand which might decrease the profitability of the store. On the other hand, if they order a new batch, in addition to the fixed cost of ordering the new batch, since the customers would prefer the fresh products, the current aged products cannot be sold and would go to waste. In addition to the timing of the inventory replenishment, the managers also need to decide about the quantity in each order and the pricing of these products over time. As the products age, the demand for the products will decrease, however by changing the prices considering the freshness of the products, the demand can be altered in order to maximize the profitability of the company. Sen [45] states that pricing decisions can be challenging since decreasing the price too early or too deeply will lead to lost revenue, while high prices might lead to lost sales and unsold items at the end of the season. We aim to determine the optimal timing and quantity of the inventory replenishment batches and the dynamic pricing strategies for perishable products considering an age-dependent stochastic demand function. The dependency of demand to the age of products makes these decisions much more complicated to analyze.

There has been a broad history of research on the management of perishable products since 1950's. Bakker et al. [6] and Janssen et al. [29] provide detailed literature reviews about the management of perishable products. However, most of the studies in the literature focus on pricing and inventory decisions separately and not many of the sources include coordinated inventory and pricing decisions.

Nahmias [41], Karaesmen et al. [31], Chaudhary et al. [11] present detailed literature reviews about inventory management of perishable products. Van Zyl [47], search the optimal inventory strategy for a perishable product that has two period life-time with a periodic review system. When products with longer lifetimes are considered, the state space of the problem increases exponentially and it becomes very hard to find optimal solutions in a reasonable time limit. If many different products with different ages are allowed to be in stock at the same time, they all need to be kept as state variables, leading to a much higher dimensional state definition and the problem becomes much more difficult to solve. In order to decrease the number of decision variables in their model, Nahmias [40] groups these variables to obtain a solvable equation form. Due to the difficulty of this problem under generalized settings, different authors analyze different inventory strategies for perishable products under varying assumptions. Berk and Gürler [8] analyze (Q, r) inventory strategy and develop heuristics by considering fixed order cost and positive lead time. Jing et al. [30] aim to determine the dynamic lot sizes for perishable inventory under minimum order quantities considering age-dependent stock deterioration rates and inventory costs. Minner and Transchel [37] analyze inventory management of perishable products considering different characteristics and service levels, and they offer dynamic ordering strategies different than (s, S) and compare these strategies with the (s, S) strategy. Haijema [22] develop a stochastic dynamic programming model to compute the optimal ordering, disposal and issuance decisions. Haijema and Minner [23] compare a number of new and existing base-stock

policies and constant order policies by determining their optimal parameter values through simulation-based optimization. Haijema and Minner [24] consider the age of the stock and provide new stock-age dependent order policies in addition to the existing ones. They discuss the value of stock-age information and present a new policy that performs close to optimal.

Most of the studies about the inventory management of perishable products assume that the demand is independent of the age or freshness of the products. However, for perishable products, as stated by Xu and Cai [51], customers prefer to buy fresher products instead of older ones. Thus, the demand is affected by the age of the products in stock. Benkherouf [7], Mishra and Singh [38], Broekmeulen and van Donselaar [10] analyze the perishable products' inventory decision process under time-dependent demand, and they suggest inventory management algorithms that consider amount and the age of the inventory. Ferguson and Ketzenberg [20] analyze the value of sharing maturity information of the products between a supplier and a retailer. Wang and Lin [49] address the optimal replenishment strategy for deteriorating products under continuous decrease in market demand and price changes. However, none of the studies above consider pricing as a decision in their model.

When we consider literature of dynamic pricing, we observe that a certain amount of lifetime is assigned to perishable products to be sold, as can be seen in airline or hotel management industries. Elmaghraby and Keskinocak [18] present a detailed literature review about dynamic pricing with inventory considerations. Transchel and Minner [46] consider coordinated inventory and pricing decisions for non-perishable products with deterministic demand. Liu et al. [36] aim to determine a joint dynamic pricing and preservation technology investment strategy for perishable foods with a deterministic demand rate that depends on price and continuously decaying quality. There are also several papers such as Rajan and Steinberg [42], Abad [1, 2, 3, 4], Wee [50], Mukhopadhyay et al. [39], P-S You [52], Sana [44], Ghosh et al. [21], Herbon et al. [28], Adenso-Diaz et al. [5], Herbon and Khmelnitsky [27], Feng et al. [19], Li and Teng [35] which investigate coordinated pricing and inventory decisions for perishable products under deterministic demand.

Considering a stochastic demand process, Chen and Sapra [12] research integrated pricing and inventory decision problem for perishable products which have two-period lifetime under periodic review. Chew-Peng and Chulung [14] have a discrete time dynamic programming model for perishable products with two-period lifetime, and observe the structural properties of the optimal solution. They also develop three heuristics to decide inventory allocation and prices of the products that extend over the two-period lifetime. Li et al. [33] also analyze the pricing and inventory replenishment decisions and develop a base-stock/list-price heuristic policy for products with arbitrary fixed lifetimes using the optimal solution structure of a two-period lifetime problem and insights from the numerical experiments. Chen et al. [13] investigate the structural properties of the optimal solution considering a fixed lead time. Chintapalli [15] analyze combined pricing and inventory control problem for perishable goods under uncertain and price-sensitive demand by classifying the products as new and old units. Chua et al. [16] consider four models for a retailer selling a perishable product with uncertain demand in order to decide whether to discount older items, how much discount to offer, and what should be the replenishment policy. In their first three models, the product is assumed to have a shelf life of two periods while their last model explores what happens with

longer shelf-life through approximation models. Li et al. [34], similar to our study, analyze the joint pricing and inventory control problem when the retailer does not sell new and old inventory at the same time, however, they consider a periodic review inventory system rather than the continuous review strategy analyzed in this study.

In this study, as commonly seen in real life examples, we consider a retailer that applies a single price to the same products, even if they have different ages, however the price can be changed over time. For example, in many stores, the same type of products have the same price at a certain time, although they have different expiration dates on them. Groceries with different freshness are commonly seen to be sold at the same price even though some of them can be clearly seen to be fresher than others. Herbon [26] compare the policy of selling a single product-age with a policy of selling multiple product-ages and prove that the optimal policy is not to sell multiple product-ages at all under certain conditions. Li et al. [34] also assume that the retailer does not sell new and old inventory at the same time and decides whether to dispose of ending inventory or carry it forward to the next period at the end of each period in addition to replenishment and pricing decisions. They also present several arguments why new and old inventory are not offered simultaneously in real life situations. Kaya and Ghahroodi [32] analyze a periodic review inventory system with dynamic pricing and compare selling single and multiple product-ages with each other, and show that the difference is very small in general.

Different than the literature, we consider integrated inventory and dynamic pricing decisions for perishable products under a stochastic age and price dependent demand function in a continuously reviewed inventory system. Our main focus about the inventory replenishment in this study is on the tradeoff between the decreased demand rate over time if the managers continue with the current stock, and the cost of wasting the current stock in addition to fixed order costs, if a new batch is ordered. Note that such a tradeoff does not exist in models in which demand is independent of the age of the products. Our model is not restricted with two-period lifetimes, as widely seen in the literature, and it includes multi-period lifetimes. We aim to decide on the optimal prices dynamically considering the state of the system at any time, in addition to the timing and quantity of the orders. For this purpose, a stochastic dynamic programming formulation is developed and certain characteristics of the optimal solution of this model are proven. We also extract certain managerial insights through numerical experiments.

2. **Model.**

We consider a single product, continuous review inventory system under dynamic pricing such that at every point in time, the state of the system denoting the amount of products in the inventory and the age of these products, are known and kept as the state variables. Depending on the state of the system, first an inventory replenishment decision is made such that either new products are ordered at that time or the system is continued with the current products at hand. We assume that the order lead time is zero and thus if new products are to be ordered, the decided order amount, Q, will be acquired immediately. We let A denote the fixed ordering cost and c denote the unit cost per product. In addition, a pricing decision is given at each state that defines the price of the products at that state.

We assume that the retailer applies a dynamic single price strategy, such that the same type of products are sold at the same price at any time, even if they have different ages, however the single price can be changed over time. As stated

by Herbon [26], customers do not prefer to buy older products when fresher ones are available in stock, if they are sold at the same price. Thus, in a continuous review inventory setting with no lead time, the older inventory can never be sold after a new batch is ordered. Even when the new batch is all depleted, another replenishment will be made at that time and the older inventory will always remain unsold. We note that products with different ages might be sold at different prices such that fresher products are more expensive and older ones are sold at a discount rate. In such a case, customer choice behaviors can be analyzed and pricing can be adjusted based on the choices of different customer segments. However, the analysis of such a system is out of the scope of this paper and we leave it as a future study.

In this study, similar to Li et al. [34], we assume that the retailer does not sell new and old products at the same time at different prices. Duan et al. [17] also state that in real life, since the perishable products deteriorate gradually by time, disposing of the remaining is much more selected, when a new batch is ordered. Thus, in our model, when new products are ordered, the older ones in stock are all assumed to be salvaged at unit price, s. As a result, all the products in the inventory will be of the same age at any time and they are sold at the same common price. When a price change is made, it will apply to all products in stock.

In our analysis, the state of the system can be defined via two state variables, denoted as (q, t), since all the inventory in stock will be of the same age, where q denotes the amount of inventory and t denotes the age of inventory. We let $p(q, t)$ denote the price of the products depending on the quantity and age of inventory on hand. Demand is assumed to arrive one by one according to a non-homogenous Poisson distributed demand function, where $\lambda(p, t)$ denotes the demand rate when the price is p and products in stock are of age t.

2.1. **Dynamic programming formulation.** We analyze the continuously reviewed inventory system by discretizing the time horizon with very small increments of length ϵ. As stated in Ross [43], a non-homogeneous Poisson process $N(t)$ with rate $\lambda(p, t)$ satisfies the following relations: (i) $N(t)$ has independent increments, (ii) $P(N(t+\epsilon) - N(t) \geq 2) = o(\epsilon)$ and (iii) $P(N(t+\epsilon) - N(t) = 1) = \lambda(p, t)\epsilon + o(\epsilon)$. These relations state that the amount of demand to happen at any time interval is independent of the previous demand occurrences, and as ϵ converges to 0, the probability of 2 or more demand to happen during a time period of length ϵ also goes to 0. Using these properties, we develop a dynamic programming formulation by dividing time into small pieces, each of length ϵ. As a result, there is a certain probability of $\lambda(p, t)$ such that one unit of demand arrives during that small time period, and the probability of having two or more demand is 0 during that period.

We develop an infinite horizon average cost dynamic programming formulation, which fits into our model considering the structure of the problem. We approach the problem by dividing the continuous time slots into little discrete pieces of length ϵ. Without loss of generality and in line with the literature, we let $\epsilon = 1$ time unit (note that this time unit can be very small such as a millisecond etc.), to apply the non-homogenous Poisson process features as explained above. In these models, we denote t as the current age of inventory, and it increases by $\epsilon = 1$, when each small amount of time is passed. Note that when a new replenishment is made, the age of the inventory, t is set to 0. The demand function is assumed to be a non-increasing function of the age of the products. In this formulation, we aim to maximize the average profit per unit time, denoted as μ, and apply the Bellman's equation (please refer to Bertsekas [9] for details about Bellman's equation for average cost dynamic

programming formulations) for this problem as below;

$$
\begin{aligned}
h(q,t)+\mu &= \max\{Continue, Order\} = \max\{f(q,t), f_0(q)\} && (1)\\
f(q,t) &= \max_p[\lambda(p,t)(p+h(q-1,t+1)) \\
&+ (1-\lambda(p,t))h(q,t+1)] \qquad \forall q>0, t<T && (2)\\
f_0(q) &= \max_Q[-A-cQ+sq+\max_p[\lambda(p,0)(p+h(Q-1,1)) \\
&+ (1-\lambda(p,0))h(Q,1)]] \qquad \forall q>0 && (3)\\
h(0,t)+\mu &= f_0(0) \qquad \forall t<T && (4)\\
h(q,T)+\mu &= f_0(q) \qquad \forall q>0 && (5)
\end{aligned}
$$

We obtain the optimal solution of this problem by solving the above equations that maximize the value of μ, where T denotes the maximum lifetime of the products. In order to solve the above equations, we let $h(0,0)=0$ as the reference state value and $h(q,t)$ denotes the relative value function at state (q,t) with respect to $h(0,0)$, as explained in detail in Bertsekas [9].

In Equation (1), we try to maximize the objective value μ, by deciding to "continue" without a replenishment or "order" new products at state (q,t). In Equation (2), at any state, if we choose to "continue", we need to decide on the price p for that state that will maximize the average profit value μ. Depending on the price p, with probability $\lambda(p,t)$ one unit of demand arrives and we obtain a price p at the current time, the inventory decreases by 1 and the future relative value function will be $h(q-1,t+1)$ when time is passed by $h=1$. With probability $1-\lambda(p,t)$, no demand arrives in that time period and we move to the new relative value function $h(q,t+1)$ in the next period. Therefore the total value equation, if we choose to continue, will be as in Equation (2).

In Equation (3), we consider "ordering" new products, in which case we need to decide on the order amount Q that will maximize the average profit value μ. In this case, a fixed order cost A and unit cost c per product needs to be paid but since the older products are salvaged at this time, an income of sq is added to this function. When new products are ordered, they will be of age 0 and a pricing decision is also need to be made for these products. For these newly ordered products, age of these products is updated to "0" and with probability $\lambda(p,0)$, one unit of demand arrives, a sale is made at price p at the current time and the future relative value function will be $h(Q-1,1)$ at the beginning of the next period. With probability $1-\lambda(p,0)$, no demand arrives and the future relative value function will be $h(Q,1)$.

Equations (4) and (5) denote the boundary conditions of this model, such that if the amount of inventory is depleted to 0, a replenishment needs to be made at that time, or if the maximum lifetime of the products is achieved without being sold, they need to be salvaged and a new replenishment again needs to be made.

2.2. **Analysis of the optimal solution.** In Theorem 2.1, we prove our main result related to the optimal solution of the above model that states that the ordering decision will be based on only a time threshold, independent of the inventory amount at hand:

Theorem 2.1. *Under the model assumptions given above, there exists a time threshold τ such that, new products are ordered when the age of the products at hand reaches τ, or when the quantity drops to 0.*

Proof. Using the DP equations in (1)-(3)

$$\begin{aligned} h(q,t)+\mu &= \max\{f(q,t), f_0(q)\} \\ &= \max\{\max_p[\lambda(p,t)(p+h(q-1,t+1))+(1-\lambda(p,t))h(q,t+1)], f_0(q)\} \end{aligned}$$

Recall that new products should be ordered when the age of the products reaches T, i.e. for all q, $h(q,T) = f_0(q) - \mu$. Using induction, for $t = T-1$ and for all $q \geq 1$:

$$\begin{aligned} h(q,T-1)+\mu &= \max\{\max_p[\lambda(p,T-1)(p+f_0(q-1)-\mu) \\ &+ (1-\lambda(p,T-1))(f_0(q)-\mu)], f_0(q)\} \\ &= \max\{\max_p[\lambda(p,T-1)(p-s)+f_0(q)-\mu], f_0(q)\} \\ &= f_0(q)+\max\{\max_p[\lambda(p,T-1)(p-s)-\mu], 0\} \end{aligned}$$

Thus, the optimal decision at time $T-1$ is independent of the quantity at hand (as long as $q > 0$), and it is to order new products if $\max_p[\lambda(p,T-1)(p-s)-\mu] \leq 0$ and as a result $h(q,T-1) = f_0(q) - \mu$. If $\max_p[\lambda(p,T-1)(p-s)-\mu] > 0$, the optimal decision is to continue for at least one more time period with the current inventory.

If the optimal decision at time $T-1$ is to order new products, we can apply the same reasoning above for $t = T-2$ and the same result follows for $t = T-2$. Since we assume that the demand is a non-increasing function of age, $\lambda(p,t)$ does not increase as t increases. Thus, starting from T, the optimal decision will be to order new products for all t until $\max_p[\lambda(p,t)(p-s)-\mu] > 0$ is satisfied for some t (Note that if this relation is not satisfied for any t, then the optimal decision will be to order new products at all periods t). Let $t = \tau$ denote the first time when the optimal decision is to continue such that $\max_p[\lambda(p,\tau)(p-s)-\mu] > 0$. Then, we know that $f(q,\tau) \geq f_0(q)$ for all q, and $h(q,\tau) = f(q,\tau) - \mu \geq f_0(q) - \mu$ for all q. Also note that for all $q \geq 1$, $f_0(q) = f_0(q-1) + s$. Then, for all $q \geq 1$, the problem at age $\tau - 1$ will satisfy the following relations:

$$\begin{aligned} h(q,\tau-1)+\mu &= \max\{\max_p[\lambda(p,\tau-1)(p+h(q-1,\tau)) \\ &+ (1-\lambda(p,\tau-1))h(q,\tau)], f_0(q)\} \\ &\geq \max\{\max_p[\lambda(p,\tau-1)(p+f_0(q-1)-\mu) \\ &+ (1-\lambda(p,\tau-1))(f_0(q)-\mu)], f_0(q)\} \\ &= \max\{\max_p[\lambda(p,\tau-1)(p-s)+f_0(q)-\mu], f_0(q)\} \end{aligned}$$

Since the optimal decision is to continue at time τ, we know that $\max_p[\lambda(p,\tau)(p-s)-\mu] > 0$ and since $\lambda(p,\tau)$ is non-increasing in τ, $\max_p[\lambda(p,\tau-1)(p-s)-\mu] \geq \max_p[\lambda(p,\tau)(p-s)-\mu] > 0$ and the optimal decision is to continue at time $t = \tau-1$, too. Thus, the optimal decision is to order new products until the time threshold τ and to continue at all other time periods $t \leq \tau$, independent of the amount of inventory, q. We can define this relation as below for any t, and since $\lambda(p,t)$ is non-increasing in t, the proof is complete.

If $\max_p[\lambda(p,t)p] \geq \mu$	Decision at time t is: "No order";
If $\max_p[\lambda(p,t)p] < \mu$	Decision at time t is: "Order"

□

This result has important implications for the inventory management of these products and it mainly states that if the products at hand reach to a certain age, they need to be salvaged, independent of the quantity at hand, since there will not be enough demand rate to sell them in the future periods. Thus, instead of waiting and losing future demand, salvaging the old ones and ordering fresher products will be more profitable for the company. However, determining the optimal replenishment threshold value for the age of the products is an important and critical decision for the effective management of these products. We note that the above result does not contradict and is consistent with the classical results in the inventory management literature. For example, for durable products, when the demand rate does not depend on the age of the products (i.e. $\lambda(p,t) = \lambda(p,0)$ for all t), the time threshold will never be reached. Thus, the optimal solution will be to continue for all t (as can be seen from the relation at the end of the proof of Theorem 2.1), until q becomes 0. This result essentially denotes the base-stock policy, such that Q units are ordered when the inventory drops to 0, which is shown to be the optimal policy when the lead time is zero. When the demand rate depends on the age of the products as in this paper, we denote the policy in Theorem 2.1 as (τ, Q) policy as an extension of the base-stock policy. Note that the result in Theorem 2.1 only holds under the given model assumptions and it might not hold in different systems when, for example, a periodic review inventory system is used, lead time is non-zero or demand is allowed to arrive in batches.

When lead time (L) is deterministic but non-zero, $L > 0$, due to Theorem 2.1, replenishment decision will be made when the age of the products reaches to $\tau - L$ such that new batch will arrive to the system exactly at time τ. However, in this case, since there is a lead time, if the quantity at hand drops below a certain level $r > 0$ when the age of the products is $t < \tau - L$, it might be optimal to order a new batch at that time. Since the demand rate depends on the age of the products, the reorder level, $r(t)$, need also be a function of the age of the products, t. For example, when the products are fresh, the demand rate will be high and more products are needed to satisfy the demand during the lead time. However, as the products get older, the demand rate decreases and less products might be enough to satisfy the demand during the lead time. The reorder quantity depends on the age of the products. Thus, the optimal policy would be to order when the age of the products reaches $\tau - L$, or the quantity at hand decreases to $r(t)$, which is a function of the age of the products, whichever occurs first. Note that this policy will be equivalent to the (r, Q) policy in the classical inventory literature for durable products, when the demand rate does not depend on the age of the products. We can denote this policy as $(\tau - L, r(t), Q)$ policy as an extension of the (r, Q) policy. However, determining the optimal reorder quantity function $r(t)$ in this case is much more complicated due to the dependency of demand on the age of the products. We leave the detailed analysis of the system with a positive lead time for further studies.

3. **A special case: Analytical model under static pricing.** We note that the dynamic programming formulation stated above can be used to determine the optimal replenishment, order quantity and dynamic pricing decisions. However, as stated above, we need to divide the time horizon into very small pieces (ϵ), such that at most one sale can be made in each period in the dynamic programming formulation, which might result in a very large number of periods, making the computations

very difficult. Of course, using larger ϵ values will decrease the computation time, however, using larger ϵ values will also lead to approximate and non-exact results. Li et al. [34] also state that using a dynamic programming model for a similar problem may be difficult or tedious to handle due to the dimension of its state space and decisions types. Haijema and Minner [24] also state that "In practice, logistics managers prefer well structured policies they do understand, such as base stock policies".

In this section, using the result in Theorem 2.1, we develop an analytical model for the static price case, as a special case of the dynamic programming formulation. Note that, the result in Theorem 2.1 in the previous section allows us to develop the analytical model for the static pricing case. We aim to determine the optimal replenishment threshold age, optimal order quantity and the single price to be used throughout the cycle. Note that the modeling and the solution of the dynamic programming formulation requires us to keep track of all possible states and the use of computer programming. However, the analytical modeling approach is much simpler to solve and easier to use by the managers since it is simpler to understand and does not require as much computation.

In this model, we utilize the renewal reward approach and calculate the expected profit of a renewal cycle, considering the demand process as a "non-homogeneous Poisson process". We use the following notations in our model.

Notations:

$f_d(x,s)$: Probability of having x units of demand between time 0 and s

$N(s)$: Observed number of demand until time s

$m(s)$: mean value function for the non-homogeneous Poisson process between time 0 and s

$\lambda(p,t)$: Demand rate for price p and age t

$$\begin{aligned} f_d(x,s) &= \frac{e^{-m(s)}(m(s))^x}{x!} \\ m(s) &= \int_0^s \lambda(p,t)dt \\ P(N(s)-N(0)=n) &= \frac{e^{-m(s)}(m(s))^n}{n!} \end{aligned}$$

In this system, every time a replenishment is made, since all the older ones are salvaged, a new cycle begins with Q units of fresh products at hand. However, in each cycle, due to the random demand, different profit values can be obtained and each cycle can be of different lengths. We let π_i denote the profit obtained in cycle i and T_i denote the length of that cycle. We aim to maximize average profit per unit time over an infinite amount of time and our objective function can be shown as below, that defines the total profit obtained in n cycles divided by the total length of these cycles.

$$\begin{aligned} \max E[Profit\ per\ unit\ time] &= \lim_{n\to\infty} \frac{\pi_1+...+\pi_n}{T_1+...+T_n} = \lim_{n\to\infty} \frac{\frac{\pi_1+...+\pi_n}{n}}{\frac{T_1+...+T_n}{n}} \\ &= \frac{E[Profit\ per\ Cycle]}{E[Length\ of\ a\ Cycle]} \end{aligned}$$

where π_n= Profit in cycle n and T_n= Time spent in cycle n.

We aim to maximize the expected profit per unit time by deciding on p, τ and Q, as stated below.

$$\max_{p,\tau,Q} \frac{E[Profit\ per\ Cycle]}{E[Length\ of\ a\ Cycle]} = \frac{E[\pi]}{E[T]} \tag{6}$$

For the calculation of the expected profit per cycle and the expected length of a cycle, we need to consider two cases which might occur depending on the amount of demand between time 0 and the threshold value τ. All the inventory might be sold out before τ, at which time a replenishment will need to be made and the length of the cycle will be shorter than τ, or there might still be some unsold inventory at the replenishment time τ, and they will be salvaged at that time. Thus depending on the demand D between time 0 and τ being less than Q or not, we can write the expected profit and expected length per cycle as below:

$$\begin{aligned} E[\pi] &= E[\pi|D<Q]P(D<Q) + E[\pi|D\geq Q]P(D\geq Q) \\ E[T] &= E[T|D<Q]P(D<Q) + E[T|D\geq Q]P(D\geq Q) \\ E[T|D<Q]P(D<Q) &= \sum_{j=0}^{Q-1} \frac{e^{-\int_0^\tau \lambda(p,t)dt}(\int_0^\tau \lambda(p,t)dt)^j}{j!}\tau \end{aligned}$$

$$E[\pi|D<Q]P(D<Q) = \sum_{j=0}^{Q-1} \frac{e^{-\int_0^\tau \lambda(p,t)dt}(\int_0^\tau \lambda(p,t)dt)^j}{j!}(pj + s(Q-j) - A - cQ)$$

In the first case, Q units cannot be sold before the threshold time τ. We calculate the probability of selling j units until time τ, multiply that probability with the profit when q units are sold and sum over all j values between 0 and $Q-1$. Similarly, for the expected cycle length, if Q units cannot be sold until time τ, the length of the cycle will be τ and thus we multiply τ with the total probability of not being able to sell Q units until time τ.

In the second case, if Q or more demand arrives until time τ, all Q units will be sold before τ and the profit will be $(pQ - A - cQ)$ in that cycle. Thus, we multiply this value with the total probability that Q or more demand arrives until time τ in order to calculate the expected profit in this case.

If Q or more demand arrives before time τ, the length of a cycle will be less than τ, and it will be equal to $x < \tau$ such that Q^{th} demand arrives exactly at time x. In this case, the length of a cycle will be x if $Q-1$ units of demand arrives until time x and the next demand arrives at time x. Thus we multiply x with the probability of having $Q-1$ demand until time x, and the probability of having one unit of demand during that time period, $\lambda(p,x)$. We integrate over all x from 0 to τ and determine the expected length of a cycle in this case.

$$\begin{aligned} E[\pi|D\geq Q]P(D\geq Q) &= (pQ - A - cQ)\sum_{j=Q}^{\infty} \frac{e^{-\int_0^\tau \lambda(p,t)dt}(\int_0^\tau \lambda(p,t)dt)^j}{j!} \\ &= (pQ - A - cQ)(1 - \sum_{j=0}^{Q-1} \frac{e^{-\int_0^\tau \lambda(p,t)dt}(\int_0^\tau \lambda(p,t)dt)^j}{j!} \\ E[T|D\geq Q]P(D\geq Q) &= \int_0^\tau \frac{e^{-\int_0^x \lambda(p,s)ds}(\int_0^x \lambda(p,s)ds)^{Q-1}}{(Q-1)!}\lambda(p,x)xdx \end{aligned}$$

In order to solve the problem given in (6) and to determine the optimal values of p, τ and Q, we search over their feasible spaces through numerical computations, due to the complex features of the above functions.

4. **Numerical results.** We conduct numerical experiments with varying parameters to analyze the optimal pricing and inventory decisions and to compare the dynamic and static pricing approaches. In our base case experiment, we use the parameters $A = 10$, $c = 1$ and $s = 0.2$ with a demand function in an additive form such that the probability of demand in a unit time interval is defined as $\lambda(p,t) = 1 - bp - kt$ where $b = 0.1$ is the price sensitivity of demand and $k = 0.001$ is the freshness sensitivity of demand. We let $1/k = 1000$ as the expiration date of the product such that the demand rate is 0 afterwards.

Table 1 presents the optimal ordering and pricing decisions at different states (q,t), obtained by solving the DP equations (1)-(5). The search for the order quantity Q is made over integer values and the price values are searched with increments 0.01. The optimal order quantity, Q, under the base case parameters is found to be 30. In the table, the first value in each cell denotes the quantity to be ordered at that state and the value in the parentheses denotes the optimal price at that state.

Observe that, consistent with Theorem 2.1, new products are only ordered, each batch of size 30, when the age of the products in stock exceeds 148 time units, which denotes the threshold time independent of the amount in stock, as long as there exists at least one unit in stock. If there is at least one unit in stock and the age of the products is lower than 149, no new product is ordered at that time. When a new batch is ordered, the company has 30 units of fresh product at hand and tries to sell them at 5.49. However, as time passes, if the products are unsold, they age and their prices are seen to decrease such that the optimal price decreases to 4.36 for the products with age 148. For the same quantity, as the age of the products increases, the price is seen to decrease. Similarly, for the same age, as the number of products in stock increase, the price of the product decreases. When the age of the products exceeds 148, the probability of selling these products become too low such that obtaining a fresh batch and salvaging the current ones becomes much more profitable than waiting them to be sold.

We also present a sensitivity analysis in Table 2 for different parameter values than the base case experiment. We aim to analyze the effects of the parameters c, A, b and k on the optimal solution and employ the twice of each value in a full factorial experiment design. The first row of Table 2 denotes the optimal results for the base case experiment while the other rows present the results with the given parameter settings in the table. In the base case experiment, it is observed that 30 units are ordered in each new batch and these units are tried to be sold until their age reached 149, at which time a new batch is ordered. The initial price is 5.49 for the products in the new batch and the price is then changed dynamically over time depending on the state of the system, as seen in Table 1. The expected profit per unit time under the base case parameter settings is seen to be 1.7283.

When we double the price sensitivity of demand, we observe that the initial price is decreased and lower prices need to be set. Since the profit margin is decreased, the fixed cost becomes more dominant and higher quantities are ordered in each batch. In addition, since the ratio of the sale price to the salvage price is decreased,

TABLE 1. *Optimal Ordering and Pricing Decisions*

q/t	1	2	...	148	149	150	...
0	30 (5.49)	30 (5.49)	...	30 (5.49)	30 (5.49)	30 (5.49)	...
1	0 (5.82)	0 (5.81)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
2	0 (5.81)	0 (5.80)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
3	0 (5.80)	0 (5.79)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
4	0 (5.78)	0 (5.77)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
5	0 (5.77)	0 (5.76)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
6	0 (5.76)	0 (5.75)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
7	0 (5.75)	0 (5.74)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
8	0 (5.74)	0 (5.73)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
9	0 (5.73)	0 (5.72)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
10	0 (5.71)	0 (5.70)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
11	0 (5.70)	0 (5.69)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
12	0 (5.69)	0 (5.68)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
13	0 (5.68)	0 (5.67)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
14	0 (5.67)	0 (5.66)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
15	0 (5.66)	0 (5.65)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
16	0 (5.65)	0 (5.64)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
17	0 (5.63)	0 (5.62)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
18	0 (5.62)	0 (5.61)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
19	0 (5.61)	0 (5.60)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
20	0 (5.60)	0 (5.59)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
21	0 (5.59)	0 (5.58)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
22	0 (5.58)	0 (5.57)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
23	0 (5.57)	0 (5.56)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
24	0 (5.55)	0 (5.54)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
25	0 (5.54)	0 (5.53)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
26	0 (5.53)	0 (5.52)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
27	0 (5.52)	0 (5.51)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
28	0 (5.51)	0 (5.50)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
29	0 (5.50)	0 (5.49)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
30	0 (5.49)	0 (5.48)	...	0 (4.36)	30 (5.49)	30 (5.49)	...
...	...	...	...	...	...	...	...

it takes a much longer time for the company to salvage the older ones and order a new batch. The order time is increased to 266 because of this effect. As a result of decreased profit margins, the expected profit per unit time is decreased to 0.6037.

When the freshness sensitivity of demand is doubled, denoting faster decaying products or products with smaller lifetimes, we observe that less number of products are ordered in each batch and new batches are ordered much more quickly, since freshness becomes much more important in sales. Even though, the initial price is not affected much, we observe that the price change over time is much more significant than the base case results. The expected profit per unit time is decreased about 10% on average when k is doubled, however this decrease is not as significant as the effect of doubling b.

When we double the fixed cost, A, more units are ordered in each batch and a longer time is used until the next order. The initial price is not affected significantly,

TABLE 2. *Dynamic Price Model Results for Varying Parameters*

Unit cost(c)	Fixed cost(A)	Freshness Sens.(k)	Price Sens.(b)	Order Time	Order Quantity	Initial Price	Expected Profit
1	10	0.001	0.1	149	30	5.49	1.7283
			0.2	266	39	3.00	0.6037
		0.002	0.1	90	21	5.49	1.6067
			0.2	157	27	3.00	0.5246
	20	0.001	0.1	179	42	5.49	1.6080
			0.2	312	55	3.00	0.5251
		0.002	0.1	111	29	5.50	1.4384
			0.2	192	38	3.00	0.4171
2	10	0.001	0.1	254	28	5.99	1.3204
			0.2	486	33	3.50	0.2817
		0.002	0.1	143	20	5.98	1.2064
			0.2	273	23	3.50	0.2148
	20	0.001	0.1	286	39	6.00	1.2071
			0.2	545	47	3.50	0.2160
		0.002	0.1	167	27	6.00	1.0495
			0.2	321	32	3.50	0.1273

however the expected profit per unit time is seen to be decreased by about 10% on average, similar to the effect of doubling k.

Finally, we consider the effect of doubling the unit cost, c, denoting more valuable products, and observe that even though slightly lower quantities are ordered in each batch, much longer cycle times are utilized until a new batch is ordered since salvaging these products becomes much more costly. In addition, the sale prices of the products are also increased as a result of the increased unit cost, however the change in the sale prices are lower than the change in the unit cost. For example, with the case parameters, even though the unit cost is increased from 1 to 2, the initial sale price is only increased from 5.49 to 5.99. The expected profit per unit time is seen to be decreased by about 30% on average.

In Table 3, we present the optimal solutions under the single pricing model and compare the results of the dynamic pricing model with the static pricing one. We observe that benefits obtained by utilizing the dynamic pricing strategy is very much dependent to the system properties and the parameter values. For example, under the base case parameter setting, employing a static pricing strategy results in only 0.08% lower expected profit per unit time, compared to the dynamic pricing results. However, when all the parameter values are doubled, this difference increases up to 17%, as seen in Table 3. Thus, the dynamic pricing model can provide significant savings in profits under certain parameter settings. Employing a dynamic pricing strategy is much more beneficial for the systems in which profit margins are low, fixed costs are high and demand depends highly on prices and freshness of products. However, if the demand is not very sensitive to prices or freshness of the products or if the costs in the system are low, employing a static pricing strategy might be as good as the dynamic pricing one, and much simpler to manage.

TABLE 3. *Static Price Model Results for Varying Parameters*

Unit cost	Fixed cost	Freshness Sens.	Price Sens.	Order Time	Order Quantity	Best Price	Expected Profit	Percent Diff.
1	10	0.001	0.1	133	30	5.28	1.7269	0.081
			0.2	202	39	2.86	0.6024	0.215
		0.002	0.1	80	21	5.19	1.6037	0.187
			0.2	119	26	2.83	0.5205	0.782
	20	0.001	0.1	157	41	5.27	1.6053	0.168
			0.2	237	53	2.82	0.5224	0.514
		0.002	0.1	99	29	5.11	1.4329	0.382
			0.2	147	37	2.72	0.4085	2.062
2	10	0.001	0.1	186	28	5.78	1.3187	0.129
			0.2	252	34	3.29	0.2786	1.101
		0.002	0.1	105	19	5.73	1.2018	0.381
			0.2	144	22	3.23	0.2050	4.562
	20	0.001	0.1	207	38	5.77	1.2041	0.249
			0.2	282	45	3.25	0.2101	2.731
		0.002	0.1	124	27	5.61	1.0407	0.838
			0.2	170	30	3.12	0.1056	17.046

We also observe that similar quantities are ordered under the dynamic and static pricing strategies, however shorter cycle times are utilized under the static pricing case. The main reason for this result is that, under the dynamic pricing strategy, the company can increase the demand by decreasing the price when the products age, however, under the static pricing model, since changing the price is not an option, the company chooses to order a new batch much sooner as a response to the decreasing demand for aged products. We can also argue that the longer cycle time under the dynamic pricing model will benefit the environment since some more products can be sold in the remaining time at a lower price and lower amounts of products are salvaged leading to less food waste by the help of dynamic pricing.

When we compare the static price with the dynamic prices, we observe that the static price is lower than the initial price but higher than the final price under the dynamic pricing strategy. The company chooses an average single price for changing product freshness and demand over time.

5. **Conclusion.** We analyze the pricing and inventory strategies for perishable products under uncertain and age-dependent demand. We develop a dynamic programming model and prove certain characteristics of the optimal solution. We also present an analytical model for the static pricing case, which is easier to analyze and solve.

We prove that the timing of the optimal replenishment decision in our model depends only on the age of the products, independent of the quantity at hand, such that if the age of the products exceeds a certain threshold, a new batch should be ordered. Lower prices are seen to be charged as the age or the amount of the products in stock increase. We observe that the optimal replenishment and pricing decisions also depend heavily on the system characteristics. More products are seen to be ordered in each batch, with longer replenishment cycles, when demand is more

sensitive to price or less sensitive to the freshness of the products (i.e. products are less perishable) or when fixed order cost increases. Employing a dynamic pricing strategy rather than a static one becomes much more profitable when the costs in the system, and the sensitivity of demand to price and freshness of products are high. Dynamic pricing is also seen to increase the replenishment cycle lengths even though similar quantities are ordered in each replenishment.

In the future, instead of a single price assumption for all ages of products, a system in which different prices can be charged to products with different ages at the same time in store can be analyzed. In such systems, customer choice models and substitution effects will need to be analyzed. In addition, multiple products with substitution effects can be considered. Finally, analysis of periodic review inventory systems, systems with lead times or systems in which batch sales are allowed can be studied in the future.

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