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Admittance-based equivalent circuit modeling of multi-patch piezoelectric energy harvesting plate

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Abstract

Harvesting energy from mechanical vibration using piezoelectric material is a common method to power small electronics and batteries. Implementation of multiple piezo-patches to plate-like structures increases the power capacity and frequency bandwidth of the energy harvester since multiple patches can capture more vibrational modes of the dense plate dynamics. To optimize the harvested electrical output using advanced harvesting circuits, equivalent circuit modeling (ECM) is a useful technique for representing the entire electromechanical system in circuit-simulator-software (LTspice). The ECM technique based on finite-element (FE) simulation has been used for plate-like structures with only one single-piezo-patch-harvester in the literature. However, when multiple patches are integrated into a plate-like structure, their coupling can cause charge cancelation, resulting in complex dynamics that cannot be handled by the existing ECM methods. This paper presents three new methodologies for extracting a multi-mode ECM of the harvesters (e.g.: multi-patch piezo-harvesters integrated into a plate), using an admittance-based system identification approach utilizing FE results. The first method incorporates the ECM of multi-patch harvester into a single ECM, so called OGC. It includes a circuit branch where multiple patches have only one ECM branch for each vibration mode, which requires less equivalent circuit elements and hence less computational cost. The second and third so-called respective ground uncoupled (RGU) and respective ground coupled (RGC) ECM methods generate circuit branches for each piezo-patch and for each vibration mode, whereby their electrical connection (e.g. parallel or series) is later configured in LTspice. The RGU method provides a general multi-patch modeling approach while RGC method provides ECM parameters for each patch simultaneously when they are all connected which is the case in network harvester analysis. The ECMs of parallel multi-patch harvesters are validated by system-level FE simulations. The proposed admittance-based ECM methods are reliable for deriving the system parameters of multi-patch harvesters.

Keywords: multi-mode vibration modeling, plate-like structure, multiple piezo patch energy harvester, admittance-based approach, equivalent circuit modeling, electroelastic modeling

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1. Introduction

Harvesting energy from ambient vibration has been an attractive research area with the main motivation of powering small electronics or batteries, which eliminates the battery need or enhances the battery life. There have been different methods of converting vibration energy to electric power such as electrostatic [1, 2], electromagnetic [3, 4], magnetostrictive [5, 6], and piezoelectric [7, 8]. Piezoelectric conversion is one of the most preferred methods among others since piezoelectric devices have high power density and they are practical to implement in different geometric shapes and at different scales [9].

The majority of the research in literature has been done using a cantilever beam as a host structure because of its convenience in implementation [6, 9–11]. However, cantilever beam harvesters have disadvantages because of the large gap between their resonance frequencies, which leads power drop when the excitation frequency is slightly different from the resonance frequency and reduces the broadband power extraction capacity. Due to these drawbacks, harvesting efficiency is limited for the application where excitation frequencies have multiple harmonic frequency content.

Instead of beam-like structures, piezo-patches can be integrated into plate-like structures to harvest the vibrational energy of the host structure. The extensive use of plate structures in aerospace, automotive and marine applications facilitates convenient and compact energy harvesting implementations, as it does not require extra mass loading and volumetric occupancies. Moreover, plate-like harvesters have a larger number of vibrational modes than a typical cantilever harvester set up in a certain frequency range that contributes to broadband power extraction. There have been a few research on plate-like piezoelectric harvesters, which will be summarized as follows. An electromechanical finite element model (FEM) was developed using piezo patches integrated into the cantilever plate [12]. As a follow-up study, the model considered airflow excitation, and an electroaeroelastic FEM model was developed to harvest energy from an aeroelastic flutter [13, 14]. In [15], the FEM model of layered clamped piezoelectric plates and associated harvested circuit optimization study was presented for topology optimization. For large two-dimensional structures with moving force, an analytical expression of electrical power output was developed based on the strain fluctuation on the surface of structures [16]. Harne [17] presented a coupled electro-elastic analytical model for concurrent energy harvesting while attenuating the vibration of the panel by integrating corrugated piezo patches. Aridogan *et al* [18] derived an analytical closed-form solution for an all-clamped plate and validated the results by experiment.

As an electrical circuit of the harvesters, most of the studies considered a simple resistive load to assess the AC power output of both beam and plate-like harvesters [18, 19]. Nevertheless, to power the end electronic devices such as batteries and storage capacitors, AC voltage needs to be rectified to a stable DC voltage source [20, 21]. Further DC-DC conversion after initial AC-DC rectification is studied in [22] to maximize the power extraction to target devices. More efficient conversion is obtained using synchronized switch harvesting on an inductor (SSHI) circuit for a weakly coupled cantilever beam energy harvester [21, 23, 24]. The SSHI circuit integrated into a fully clamped thin plate piezoelectric harvester demonstrated power increase compared to the standard rectifier circuit [25].

The electromechanical model of the piezoelectric harvester can be represented using an equivalent circuit model (ECM) that allows simulating the performance of the harvester with a simple resistive load or complex nonlinear AC-DC rectifier circuits. The ECM of harvesters can be generated in circuit simulation software (e.g. LTspice) using physical circuit elements and the voltage response can be calculated using the numerical integration method. Extracting ECM parameters for multiple vibration modes was developed using the Rayleigh-Ritz approach [26] and later on coupled FEM-SPICE model is introduced [27] where mechanical and electrical responses are coupled using an external algorithm that created a computationally tedious process. A more efficient ECM identification method from the FEM of a cantilever piezoelectric harvester is proposed in [28]. In their work, the electromechanical coupling is included in FEM where there is no need for an external algorithm that processes the coupling between the mechanical and electrical response in every iteration. The ECM parameters are obtained directly from FEM, which is computationally more efficient. Later, the same approach was utilized for a fully clamped plate energy harvester where both AC and DC power estimations were validated by an analytical model [29]. Following this work, the authors developed an analytical model for multiple patch configuration and validated by experimental work [30].

Despite efforts in FEM-based ECM identification for single-patch plate harvesters [29] and analytical ECM identification for plate harvesters with multiple patches [30], literature lacks reports on FEM-based ECM identification for multi-patch energy harvester plates. Existing FEM-based ECM identification for single-patch plate harvesters lacks the technical detail needed to obtain the admittance and charge behavior of interconnected patches. Additionally, in the analytical ECM for multi-patch harvester, ECM parameters are derived using electromechanical coupling terms of each patch, obtained by integrating mass-normalized eigenfunction over respective patch areas. This method requires analytical modeling such as Rayleigh-Ritz for obtaining the modal properties of the structure along with the fact that, multiple ECM branches must be calculated for each patch to accommodate various vibrational modes, resulting in computational inefficiency when numerous modes are considered. Apart from these limitations, the researchers' attention to multiple patch energy harvester increased recently due to its modular design and network power optimization characteristics. For some network piezoelectric harvester applications, the nonlinear DC converter circuits need to be attached to the patch before resistive load therefore the individual patch ECM parameter would be necessary for simulating complex network behavior. To address these limitations of previous works and respond to the

need for the multiple patch ECM model we propose a single overall ECM model and multiple patch model consisting of respective ECM of each patch.

For this purpose, a multi-modal ECM of multiple piezo-patch harvesters integrated into a plate-like host structure is developed using an admittance-based system identification approach. The presented method aims to determine the multi-mode ECM parameters from the FEM of the electromechanical system where piezo-patches are located at different locations observing different modes of the structure. Three different approaches, namely, 'Overall Ground Coupled (OGC)', 'Respective Ground Uncoupled (RGU)' and, 'Respective Ground Coupled (RGC)' ECM identification methods are proposed. In the OGC method, an overall single ECM is generated from the FEM model where the patches are connected in parallel. In the RGU method, ECM is obtained from each patch individually and later they are connected parallel in LTspice. In the RGC method, respective ECM parameters are identified when all patches are interconnected and electromechanically coupled. The accuracy of each approach is evaluated based on the AC-Voltage response of the obtained ECM parameters using the system-level FEM results. Once ECM parameters are validated using AC response, the model is also valid for more complex circuit extensions such as rectifier which generates DC output [30].

The contribution and effectiveness of the proposed methods areas follows:

- The OGC method provides FEM modeling to identify the single overall ECM parameters of multiple patch harvesters for the first time, a feature not presented in previous works [29, 30].
- The RGU method offers FEM to obtain a multiple patch model when only a single patch is available. It serves as a benchmark for experimental studies, especially when there are constraints on patch availability or when impedance analyzers are limited in their ability to measure the admittance of multiple patches due to channel limitations.
- The RGC method represents the FEM version of the analytical model presented by Aghakhani *et al.* This method enables the calculation of multiple patch ECM parameters without requiring analytical modeling. It introduces a new connection configuration to obtain ECM parameters for each patch while mimicking the parallel connection configuration, essential for network configurations where all patches are interconnected. In the absence of analytical knowledge, this approach facilitates the calculation of harvester ECM parameters.

The paper is outlined as follows. First, the Single Patch ECM identification method is summarized as a baseline of the presented methods. Next, the three proposed methods are introduced by presenting FEM schematics and corresponding circuit representations, admittance, and charge responses, as well as the ECM schematics for the LTspice models. Then, the voltage response of each method is compared with the system level-finite element (FE) voltage response. Finally, the accuracy of the methods is discussed in detail.

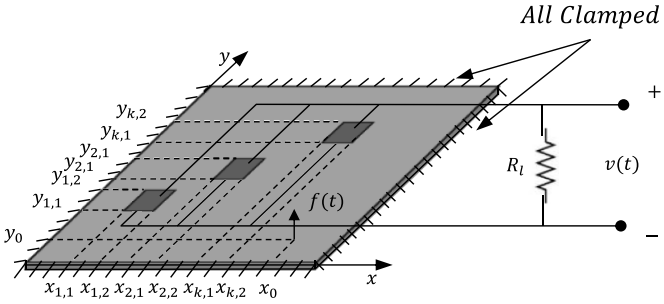


Figure 1. Piezoelectric energy harvester attached to a thin plate (all clamped) excited by transverse point force.

Table 1. Geometric and material properties.

Property	Aluminum	Piezoceramic
Length (mm)	580	72.4
Width (mm)	540	72.4
Thickness (mm)	1.96	0.267
Young's modulus (GPa)	70	69
Mass density (kg m^{-3})	2700	7800
Piezoelectric constant e_{31} $\left(\mathrm{C\ m^{-2}}\right)$	—	−190
Permittivity constant $\bar{\in}^{S}_{33}$ (nF m^{-1})	—	9.57
Damping ratio- ζ_{mn}	0.01	0.01

2. Finite element model and electromechanical identification of MPEH

2.1. Multiple patch piezoelectric energy harvester (MPEH) finite element model

The multiple piezoelectric patch harvester model used in this study is presented in figure 1. These patches are bonded to host plate, which is subjected to clamped boundary conditions on all four edges (CCCC).

A transverse point force $f(t)$ is applied at the position coordinates (x_0, y_0) to provide the mechanical vibration to host structure. Piezo patches are connected in parallel and each of them has a length of $(l_p)_k$ and a width of $(w_p)_k$ which has a corners locations at $(x_{k,1}, y_{k,1})$ and $(x_{k,2}, y_{k,2})$ where $k = 1, 2 \ldots, n_p$ and n_p represents the total number of patches attached to the host structure. The geometric and material property of the harvester is given in table 1. An output resistance (R_l) is attached between electrodes of patches as an electrical load.

The harvester model is simulated using a FEM (COMSOL) software where the harvested voltage is computed in response to the excitation force. The voltage output obtained from the harvester via FEM serves as the 'system-level' reference for validating the proposed methodologies

The coupling between the piezo-patch and the host structure is implemented by 'Form Union' command within COMSOL. This command facilitates boundary conditions that establish a relationship among the nodes located at the interfaces of bonding. A terminal and a ground are defined for positive and negative electrodes of the piezo-patches. A terminal is used in external coupling mode where a resistor is defined as

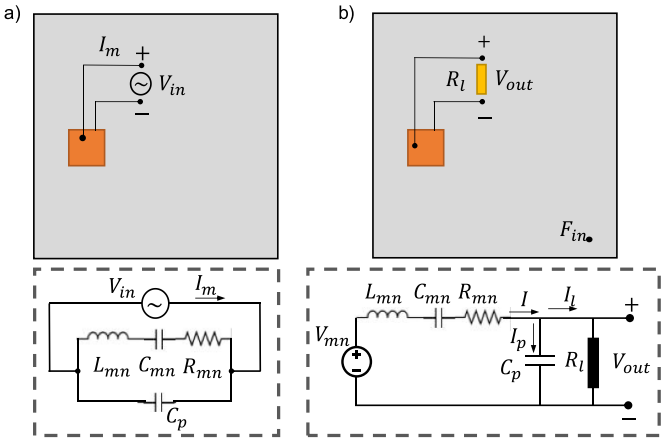


Figure 2. FEM schematics and corresponding circuit representations of single patch ECM identification method (a) for obtaining the admittance of the three piezo patches for ECM calculation, (b) for obtaining system-level voltage FRF to a force excitation.

an Electrical Circuit domain component. The resistor is linked at one end to the upper electrode and at the other end to the lower electrode, facilitating both-way electromechanical interaction and enabling the measurement of voltage output.

2.2. Single piezo-patch harvester ECM calculation procedure

The parameters of the ECM are derived from FEM simulations using a methodology similar to that detailed by Yang and Tang [28] in their study of a two-dimensional plate.

Figure 2(a) illustrates the FEM model and corresponding ECM representation to identify the L_{mn}, C_{mn}, R_{mn} parameters.

Figure 2(b) shows the FEM schematic and circuit representation used for the equivalent voltage source (V_{mn}) calculation. The piezo-patch center is placed at $x_{1,c} = 172, y_{1,c} = 272\,\text{mm}$ where subscript c represents the center point of piezo-patch.

In order to obtain the total admittance of the piezoelectric energy harvester from the FEM, the inverse piezoelectric effect needs to be considered where a harmonic voltage is applied across the electrodes of piezo patches. Then resulting current is obtained and total admittance Y can be calculated. The total admittance can also be written as

$$Y = Y_{\text{d}} + Y_{\text{mot}} \tag{1}$$

where Y_d and Y_{mot} are damped admittance and motional admittance, respectively. Y_d can directly be obtained from FE for each mode as

$$Y_{\text{d}} = \text{Im}\left[Y(\omega_{mn})\right] \tag{2}$$

and Y_{mot} is calculated by subtracting Y_d from total admittance Y. Having Y_{mot} calculated for each vibration mode, C_{mn}, L_{mn}, R_{mn} can be determined as:

Table 2. Analogy between electrical and mechanical components of the electromechanical energy harvester.

Equivalent circuit parameters for the *mn*-the vibration mode	Mechanical counterparts
Voltage source: $V_{mn}(t)$	Modal point force:f_{mn}
Inductance: L_{mn}	1
Resistance: R_{mn}	$2\zeta_{mn}\omega_{mn}$
Capacitance: C_{mn}	$1/\omega_{mn}$
Electrical charge: q_{mn}(t)	Modal time response: $\eta_{mn}(t)$
Electrical current: i_{mn}(t)	$\text{d}\eta_{mn}(t)/\text{d}t$

$$\begin{aligned} R_{mn} &= \frac{1}{\max\left[\text{Re}\left(Y_{mot}\right)\right]} \\ L_{mn} &= \frac{R_{mn}}{\omega\left[\min\left[\text{Im}\left(Y_{\text{mot}}\right)\right]\right] - \omega\left[\max\left[\text{Im}\left(Y_{\text{mot}}\right)\right]\right]} \\ C_{mn} &= \frac{1}{\omega_{mn}^2 L_{mn}} \end{aligned} \tag{3}$$

and equivalent voltage source V_{mn} of the excitation force can be calculated using the following equation,

$$V_{mn} = R_{mn} j\omega_{mn} Q\left(j\omega_{mn}\right) \tag{4}$$

where $Q\left(j\omega_{mn}\right)$ is the induced charge at each natural frequency ω_{mn} when the harvester is short-circuited. In the FEM, the resistive load is added as an electric circuit component and it is set to 1 ohm representing the short-circuit condition. The induced charge $Q\left(j\omega_{mn}\right)$ as a result of applied force is obtained using a boundary probe allowing to integrate charges on the surface of the piezo patches for each frequency step. Table 2 shows the mechanical counterpart of the ECM parameters.

The voltage response of a piezoelectric harvester obtained from system-level FEM and LTspice simulation is presented in figure 3. Both responses show perfect agreement for the modes that show resonance behavior. Note that the third and fourth vibration modes are not captured by the ECM method, because the piezo patch lies on the nodal line (see figure 4 for the location of the single patch and the mode shapes). Therefore, a single patch cannot capture all the modes, and multiple patches are required. Three different MPEH ECM identification configurations will be introduced in the following sections.

3. Multi-patch ECM calculation procedure

Two-dimensional plate-like harvesters have many vibrational modes distributed in the frequency domain. To generate a high-capacity model of these harvesters, all possible modes must be included in the ECM model. However, a single piezo-patch can capture only a certain number of the modes based on the location where it is placed and it would not be possible to capture all of the modes of a plate structure with only one single patch. Figure 4 illustrates the mode shapes of the harvester plate and it can be easily observed that the location of the patch coincides with a nodal line at the third and the fourth modes and they cannot be observed as shown in figure 3. To overcome this limitation, three identical piezo-patches

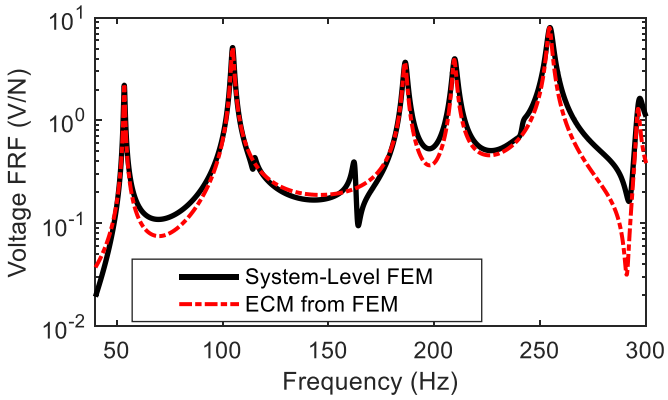


Figure 3. Single patch voltage FRF of FEM system-level simulation, and LTspice simulation based on ECM parameter obtained from FEM.

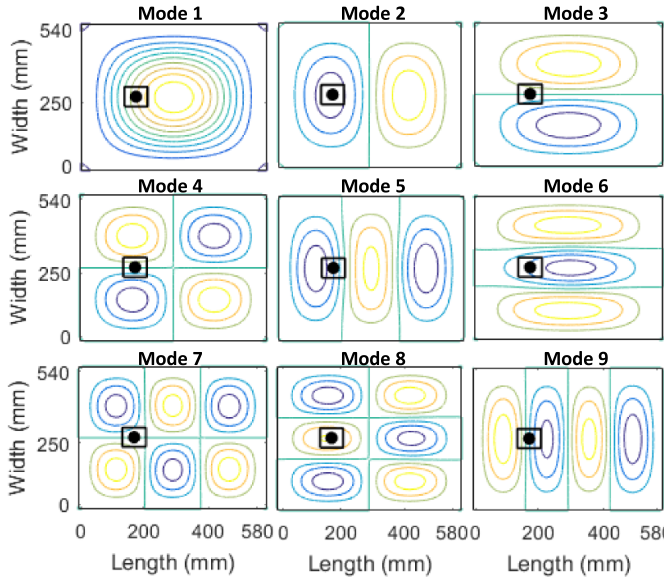


Figure 4. Mode shapes of a piezoelectric harvester based on Rayleigh-Ritz method with a single patch (left patch) location indicated on. The vibrations modes 3, 4, 7 and 9 are not captured from the piezo-patch point of view due to charge cancelations.

are utilized in the characterization of the electromechanical properties of the harvester such that the modal information included in the model is improved by the existence of the two patches.

Three different methods are proposed in the characterization of the electromechanical properties. In the first method, piezo patches are connected in parallel, and the corresponding ECM branch is obtained for each mode. In the second method, three patches are attached to the host structure unconnected, and ECM parameters for each patch are obtained individually. In the third method, all three patches are attached to the host structures connected in parallel and the ECM parameters are obtained while all patches are active. The details about the

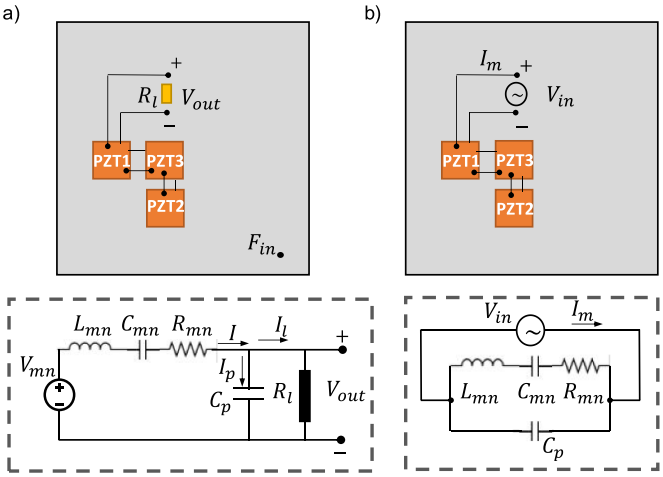


Figure 5. FEM schematics and corresponding circuit representations of OGC ECM identification method a) for obtaining system-level voltage FRF to a force excitation under resistive load and charge response under short circuit condition, b) for obtaining the admittance of the three piezo patches for ECM calculation.

implementation of these methods and their advantages will be discussed in the following sections.

3.1. Overall ground coupled (OGC) ECM identification of MPEH

In this section, three piezo patches are connected in parallel and placed at $x_{1,c} = 172\,\text{mm}, y_{1,c} = 272\,\text{mm}$, $x_{2,c} = 274\,\text{mm}, y_{2,c} = 167\,\text{mm}$ and $x_{3,c} = 274\,\text{mm}, y_{3,c} = 273\,\text{mm}$. Figure 5(a) presents the FEM schematic of the energy harvester excited by a harmonic point force F_{in} located at $x = 85\,\text{mm}, y = 85\,\text{mm}$ from the right corner. The AC voltage FRF of a harvester is obtained across a resistive load of 10 k ohm connected between the electrodes of piezo patches, which will be used as a reference to validate the ECM model. Figure 5(a) presents the ECM representation of the harvester. The admittance of the three-patch configuration is obtained using the FEM model presented in figure 5(b) where excitation voltage is applied across the electrodes of the piezo-patches. The resulting current is used to determine the admittance of the three piezo-patches. The circuit representation of the setup is shown in figure 5(b). The commercial impedance analyzer devices that can provide ECM parameters of piezoelectric materials use the same circuit representation. Developing the same measurement technique in the FEM software allows extracting the ECM parameters when there is no access to such devices.

The absolute (Y_{abs}), real (Y_{real}) and imaginary (Y_{imag}) values of total admittance obtained from the FEM is presented in figure 6. It can be observed that for each resonance frequency, there is a peak in the admittance value. However, due to the increasing trend of the admittance, some resonance frequencies do not show up in Y_{abs}. Therefore modes are captured based on the Y_{real} since it is easier to observe all the peaks for every resonance frequency. Captured modes are circled in blue color in figure 6.

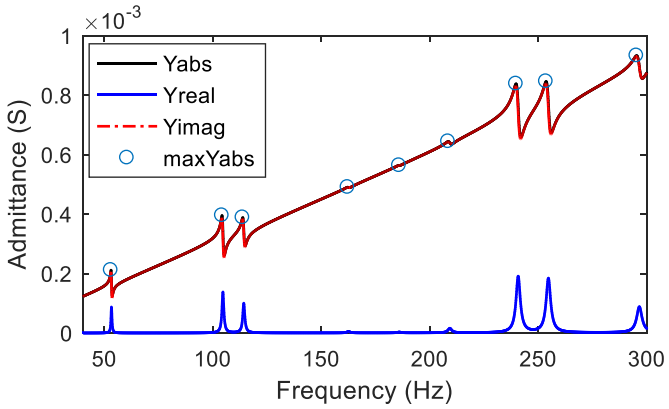


Figure 6. Admittance of the piezo harvester for OGC ECM identification method obtained from the finite-element model by applying a harmonic voltage input and measuring the resulting current.

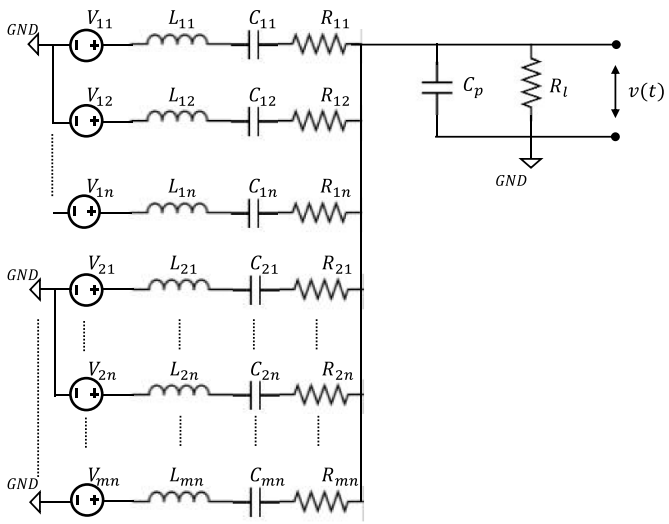


Figure 7. OGC multi-mode equivalent circuit representation of a piezo-patch harvester attached to a thin plate excited by harmonic point force to generate AC voltage across the shunted resistive load.

An eigenfrequency analysis is performed to determine the short-circuit resonance frequencies (ω_{mn}) using the FEM presented in figure 5(a) by setting the resistive load to 1 ohm. Then, using ω_{mn} and total admittance (Y), motional admittance (Y_{mot}) is obtained using equations (1) and (2).

Once the Y_{mot} is obtained, ECM parameters C_{mn}, L_{mn}, R_{mn} can be determined. Having the ECM parameters identified, the equivalent voltage source (V_{mn}) can be determined using FEM in figure 5(a) by setting the resistive load to 1 ohm to facilitate short circuit conditions. The induced charge corresponding to a 1 N harmonic point force and short-circuit resonance frequency obtained from eigenfrequency analyses are used to calculate the voltage source term.

Having the voltage source determined, the ECM can be generated in the LTspice software to simulate the AC Voltage FRF of the three-patch harvester. The schematic of the multimode ECM is illustrated in figure 7. The resistive load

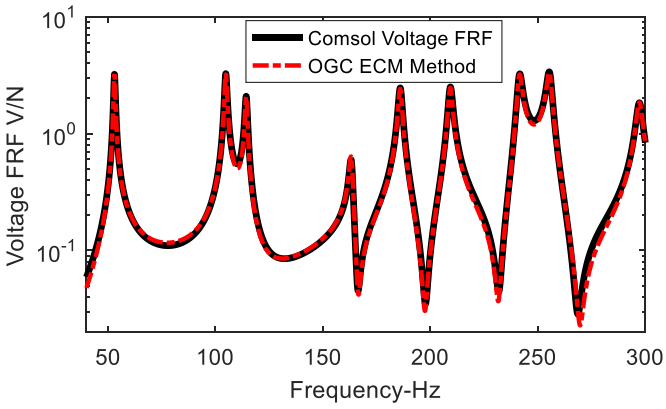


Figure 8. Three-Patch Voltage FRF (a) FEM system-level simulation, (b) LTspice simulation based on ECM parameter obtained from FEM using OGC ECM identification method.

in the LTspice model is set to 10 k ohm as in the FEM model.

For the validation of the ECM parameters, the AC Voltage FRF obtained from FEM and the one obtained from the LTspice software using the ECM parameters are compared in figure 8.

As seen in figure 8, OGC ECM method response show almost perfect agreement with the system-level FEM response for resonance frequencies and voltage amplitude of all the modes.

In the case where more than three piezo patches are attached to the structure, this method can be used considering the configuration in figure 5. The method provides a single OGC ECM model for all the patches attached on the structure. With this feature, the method incorporates the contribution of each patch into a single ECM model, which reduces the computational effort while increasing the model accuracy.

3.2. Respective ground uncoupled (RGU) ECM identification of MPEH

In this method, three piezo patches are attached to the host structure without a connection from one to other. Piezo-patches are used separately such that when the excitation voltage is applied to PZT-1 for reading the current and obtaining the admittance, PZT-2 and PZT-3 is set to the short-circuit mode such that it does not have any electromechanical effect on the harvester. The same process is applied to PZT-2 and PZT-3 while setting the other patches (PZT1-PZT3 and PZT1-PZT2) to the short-circuit model. Figure 9 demonstrates the FEM model and circuit schematic for this configuration.

With this configuration, one can obtain the admittance response over the frequency range using a boundary probe defined on the active piezo-patch surface, as shown in figure 10.

ECM parameters $C_{mn,1}$, $L_{mn,1}$, R_{mn_1}, $C_{mn,2}$, $L_{mn,2}$, R_{mn_2} and $C_{mn,3}$, $L_{mn,3}$, R_{mn_3} can be calculated based on the admittance of the three patches individually. Following the procedure explained in the previous section, an equivalent voltage

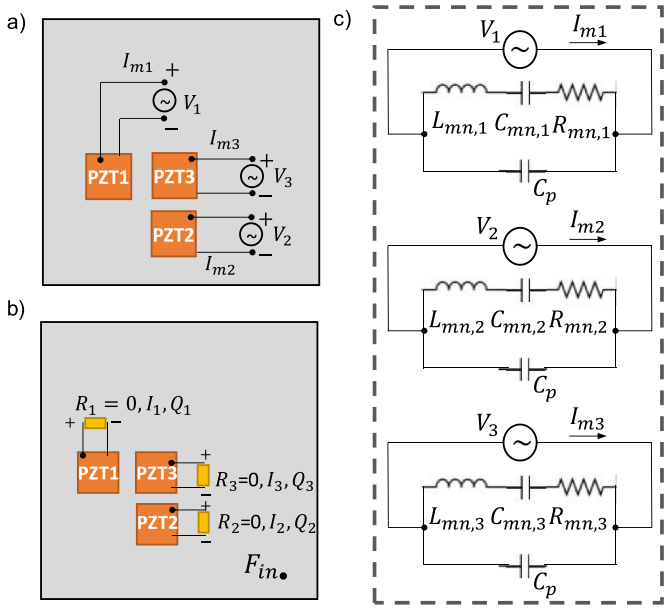


Figure 9. FEM schematics of RGU ECM identification method, (a) for obtaining the admittance of each PZT for ECM calculation, (b) for obtaining charge response of each patch under short circuit condition, (c) equivalent circuit representations of FEM used for admittance response.

source can be determined using the charges induced at each piezo patch under the effect of unit point force.

Having determined ECM parameters, the ECM model for the multipatch harvester can be generated in the LTspice software as shown in figure 11.

For the validation of the ECM model generated for each individual piezo-patch, AC FRF is generated for an output resistance of 10k ohm. Figure 12 shows AC voltage FRF obtained from FEM when piezo-patches are connected in parallel and AC Voltage FRF based on the ECM generated from the FEM model considering the individual piezo-patch procedure explained in this section.

As can be observed in figure 12, the responses show good agreement.

3.3. Respective ground coupled (RGC) ECM identification of MPEH

In this approach, three piezo patches are attached to the host structure and interconnected in parallel. To obtain the admittance response of each patch under these connected conditions, an excitation voltage is simultaneously applied to all patches via separate terminals. The excitation voltage's phase and magnitude are maintained consistently to simulate a scenario where an identical voltage is applied to all patches. This enables reading the current and the acquisition of the admittance response for each patch separately when they are electromechanically coupled. Figure 13 demonstrates the FEM model and circuit schematic for this configuration.

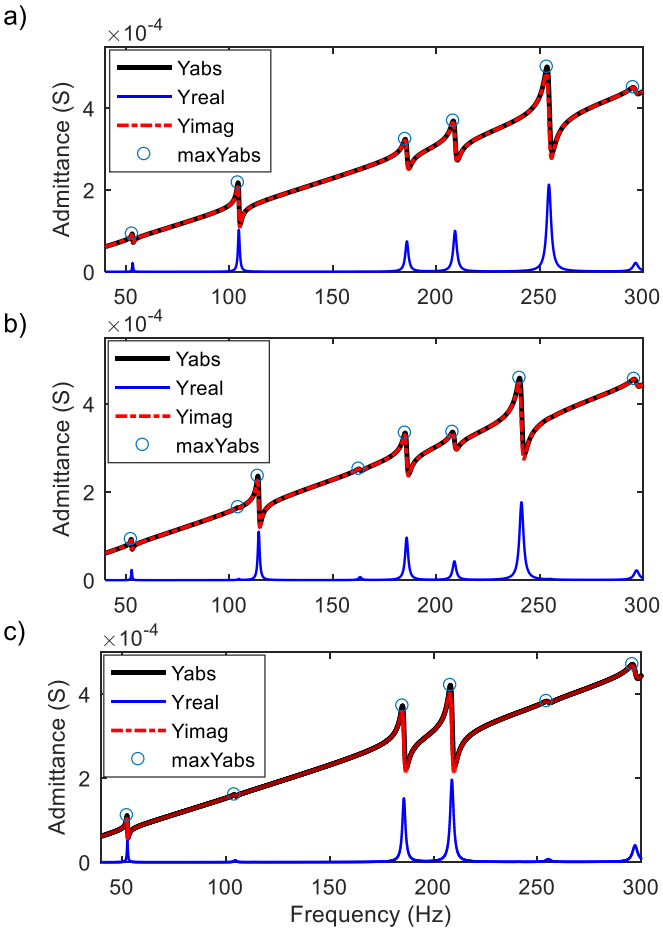


Figure 10. Admittance of the piezo harvester for RGU ECM identification method obtained from the finite-element model by applying a harmonic voltage input and measuring the resulting current (a) PZT-1, (b) PZT-2 and (c) PZT-3.

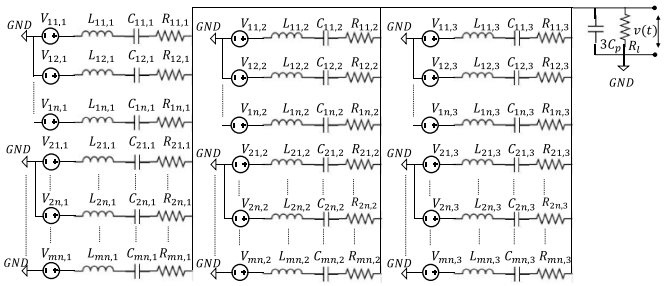


Figure 11. RGU (similarly for RGC) multi-mode equivalent circuit representation of a piezo-patch harvester attached to a thin plate excited by harmonic point force to generate AC voltage across the shunted resistive load.

With this configuration, one can obtain the admittance response over the frequency range using a boundary probe defined on each piezo-patch surface , as shown in figure 14.

ECM parameters $C_{mn,1}$, $L_{mn,1}$, R_{mn_1}, $C_{mn,2}$, $L_{mn,2}$, R_{mn_2} and $C_{mn,3}$, $L_{mn,3}$, R_{mn_3} can be calculated based on the admittance of the three patches individually. Following the

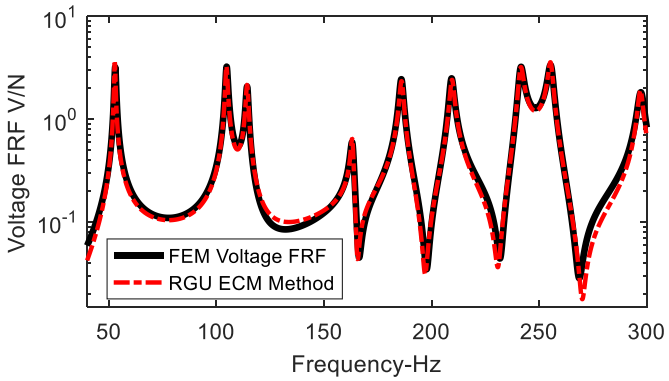


Figure 12. Three-patch voltage FRF FEM system-level simulation, LTspice simulation based on ECM parameter obtained from FEM using RGU ECM identification method.

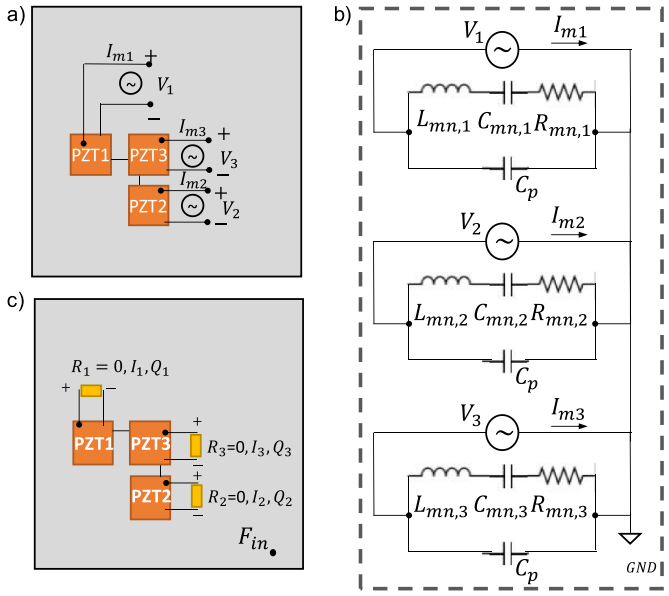


Figure 13. FEM schematics of RGC ECM identification method, (a) for obtaining the admittance of each PZT for ECM calculation, (b) for obtaining charge response of each patch under short circuit condition, (c) equivalent circuit representations of FEM used for admittance response.

procedure explained in the previous section, an equivalent voltage source can be determined using the charges induced at each piezo patch under the effect of unit point force.

Having determined ECM parameters, the ECM model for the multipatch harvester can be generated in the LTspice software as shown in figure 11.

To validate the ECM model for individual piezo-patches, we generated AC FRF using an output resistance of 10k ohms. Figure 15 illustrates the comparison between AC voltage FRF obtained from FEM simulations when piezo-patches are connected in parallel and the corresponding FRF based on the ECM model obtained using RGC method.

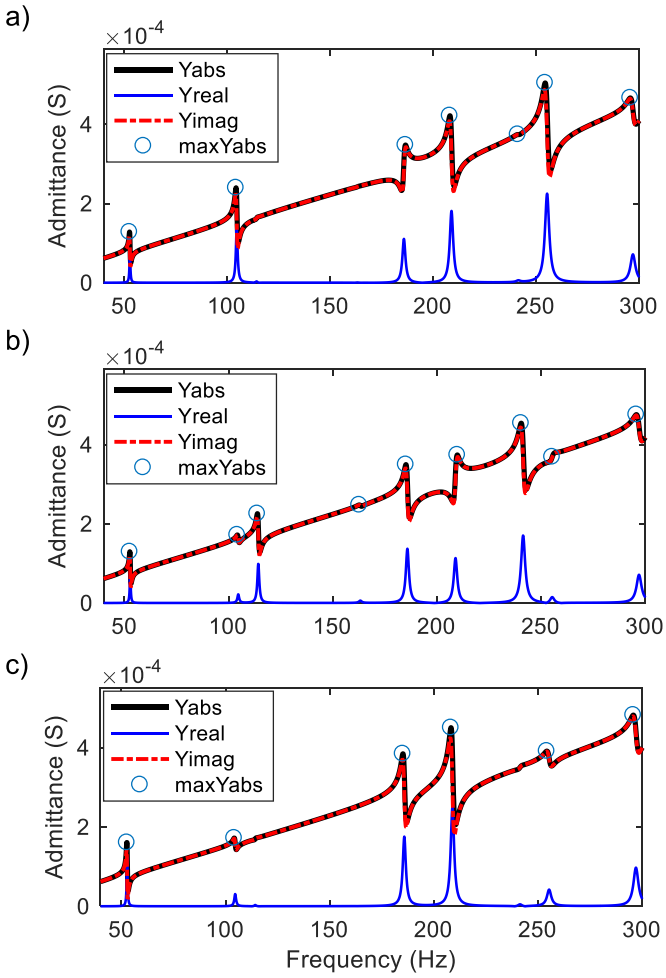


Figure 14. Admittance of the piezo harvester for RGC ECM identification method obtained from the finite-element model by applying a harmonic voltage input and measuring the resulting current (a) PZT-1, (b) PZT-2, (c) PZT-3.

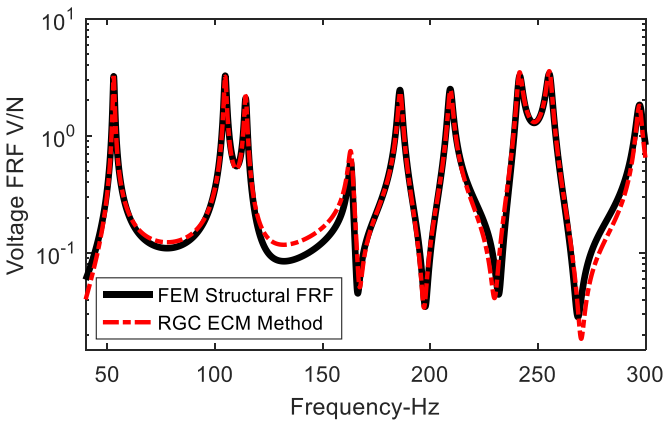


Figure 15. Three-patch voltage FRF FEM system-level simulation and LTspice simulation based on ECM parameter obtained from FEM using RGC ECM identification method.

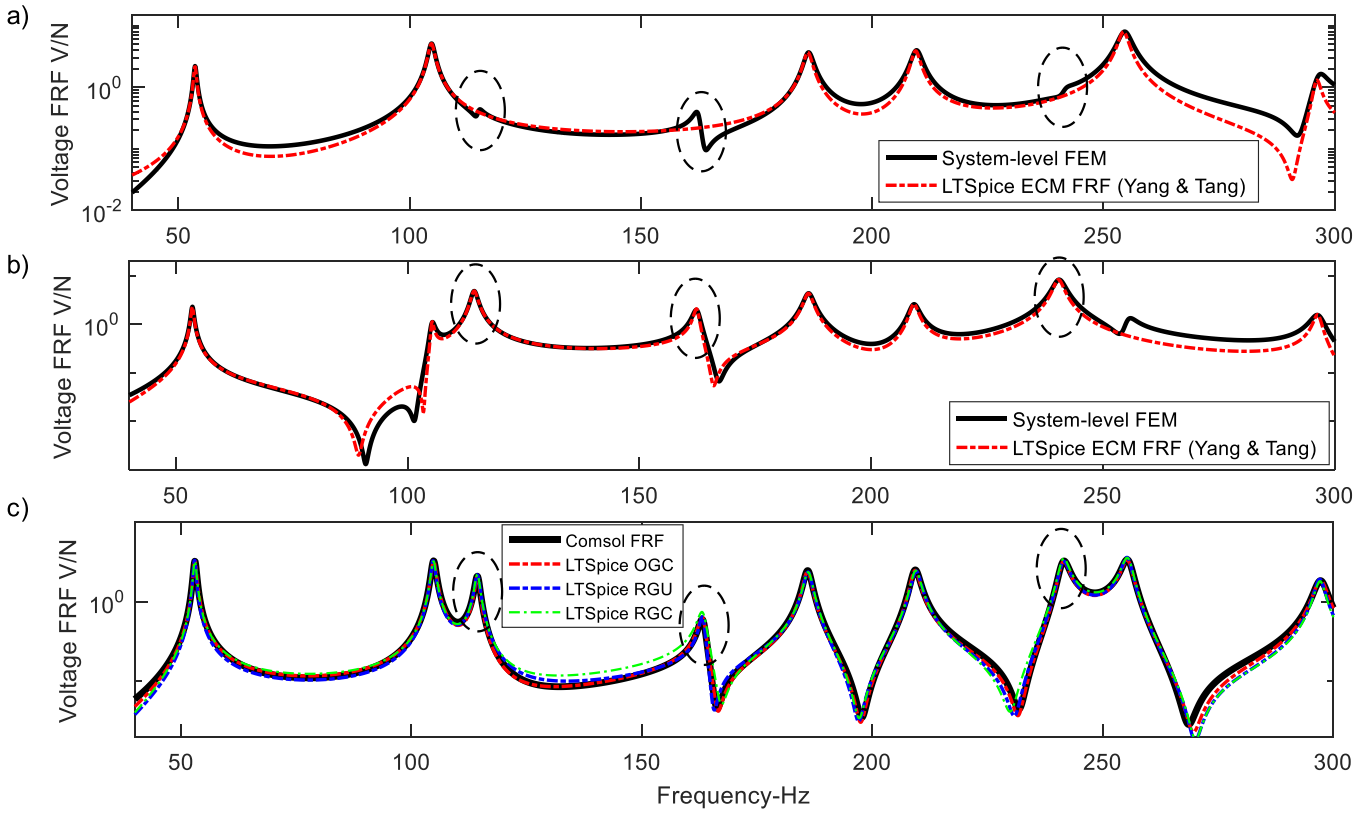


Figure 16. Voltage FRF of the piezoelectric harvester for system-level FEM and LTspice Model based on ECM approaches (a) Left Patch missing modes are circled by a dashed line, (b) Bottom Patch (PZT2), (c) Three Patch Parallel.

As can be observed in figure 15, the responses show good agreement.

4. Discussion

Identification of electromechanical characteristics of a plate harvester using the ECM method is an accurate tool for analyzing the voltage and power capacity of the harvester even with nonlinear circuit interfaces such as standard rectifiers, SSHI circuits, etc. Two-dimensional plate-like piezoelectric energy harvesters have dense modal behavior and multiple patches are required to capture the whole dynamics and represent the harvester accurately. Figure 16 summarizes the studies performed in the previous sections and compares the accuracy of each method.

As a benchmark study the results obtained with Yang & Tang model [28] is utilized for single patch validation. As seen in figure 16(a), a single piezo patch (Left Patch-PZT1) captures seven modes out of the first 9 modes with perfect accuracy. Nevertheless, mode numbers 3, 4 and 7 are not represented in the response since at these resonance frequencies the piezo patch location lies on the nodal line. In figure 16(b), bottom patch-PZT2 is employed as single patch and mode numbers 2 and 7 is not represented in the model. In figure 16(c), with three piezo-patch-model, all the vibrational modes out of the first 9 modes of the plate-like harvester are identified with high accuracy. When comparing three methods, all methods predicts the reference system-level FEM response for both resonance and anti-resonance frequencies precisely. As discussed in the previous sections, the OGC approach is computationally efficient in terms of LTspice simulation times whereas the RGU approach allows the identification of multi-mode ECM when a single patch is available for use. Moreover, the RGC approach is beneficial when multiple piezo patches are used in different configurations considering the implementation of various circuit elements to each patch. Li *et al* [31] presented different topologies for MPEH where the SSHI circuits are connected to each patch to assess the performance of the proposed topologies. In order to evaluate the performance of the networked configurations where the various circuit elements are attached to each piezo patch, individual ECM parameters for each piezo patch are required. Therefore, extracting individual ECM for each piezo patch as in the second method provides the necessary ECM parameters to analyze the performance of such a complex network of multiple piezo patch energy harvesting systems. Table 3 presents the comparison of the all methods.

5. Conclusion

This paper introduces a new and efficient methodology that includes a multi-modal characterization of the electromechanical dynamics of a plate structure integrated with multiple piezo patches. The methodology can generate the admittance-based ECM of the harvester. Three different

Table 3. Comparison of the proposed methods and previous studies.

Methods	ECM Capability	Performance/Accuracy	Advantages/Disadvantages
OGC	Single ECM for MPEH	Efficient/high accuracy	Capture the modal characteristic in a single equivalent ECM
RGU	Respective ECM for MPEH	Less efficient/high accuracy	Can be implemented in experiment
RGC	Respective ECM for MPEH	Less efficient/high accuracy	The identified ECM can be used for network MPEH
Analytical [30]	Not capable for OGC and RGU[a]	Less efficient/high accuracy	Requires prior analytical modeling knowledge
FEM Model [29]	Single patch only	N/A	No scheme for MPEH

[a] It complements analytical models by incorporating the mechanical properties of all patches when identifying the ECM parameter of the specific patch of interest.

approaches namely 'OGC', 'RGU' and 'RGC' ECM identification methods for MPEH were proposed. The OGC ECM identification approach accounted for the electromechanical interaction of three patches in the admittance response of the FEM model, whereas the RGU and RGC ECM identification methods incorporated the electromechanical contribution of each patch in the LTspice model. The proposed ECM identification approaches showed good agreement with the system-level FEM results, demonstrating their capability to include complex dynamics due to the electromechanical interaction of the multiple patches. Additionally, all approaches were developed regardless of the number of patches; therefore, the number of patches integrated into the plate could be safely increased to the desired number if the geometry of the structure gets more complicated.

The OGC ECM method can provide a computationally efficient model, especially when many modes and many patches are required to represent the harvester dynamics. On the other hand, the RGU ECM identification method can provide an accurate ECM tool when just one single patch is available, and it needs to be used at various locations to capture the whole system dynamics. To complement the second method, when multiple patch is employed on the host structure simultaneously, RGC method can be used to identify ECM parameter of each patch separately which will allow to build a network piezo electric harvester (PEH) configuration where each patch has complex nonlinear circuit extension.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Acknowledgments

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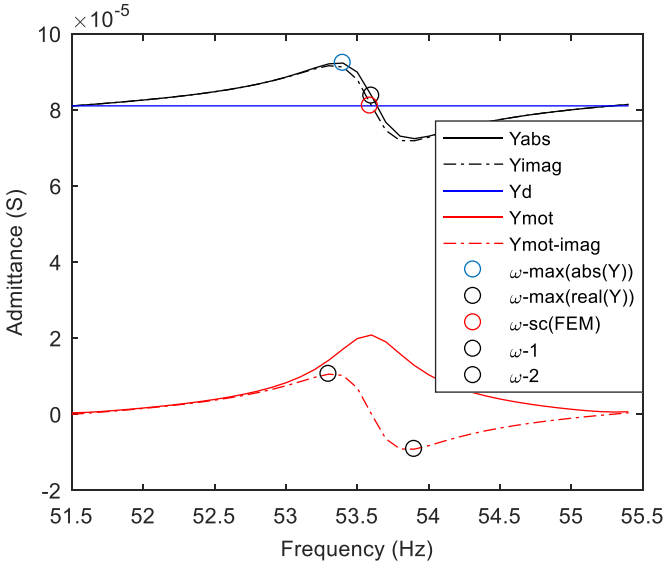


Figure 17. Total, motional and damped admittance curves and max admittance, max real admittance, short-circuit, and half power frequencies for single patch harvester.

Appendix

Figure 17 presents the total admittance (Y), damped admittance (Y_{d}), motional admittance (Y_{mot}), max total admittance frequency ($\omega_{\max(Y)}$) and max real admittance frequency ($\omega_{\max(Y_{\mathrm{real}})}$), short-circuit frequency (ω_{sc}) and half-band frequencies (ω_1 , ω_2) for the first mode. As it can be observed from figure 8 that short-circuit resonance frequency is different than the frequencies corresponding to max total admittance.

The induced charge corresponding to a 1 N harmonic point force for the first vibrational mode is shown in figure 18. Short-circuit resonance frequency obtained from eigenfrequency analyses is labeled as a red circle.

The induced charge on the piezo patches corresponding to a 1 N harmonic point force for the first vibrational mode of in RGU ECM identification method is shown in figure 19.

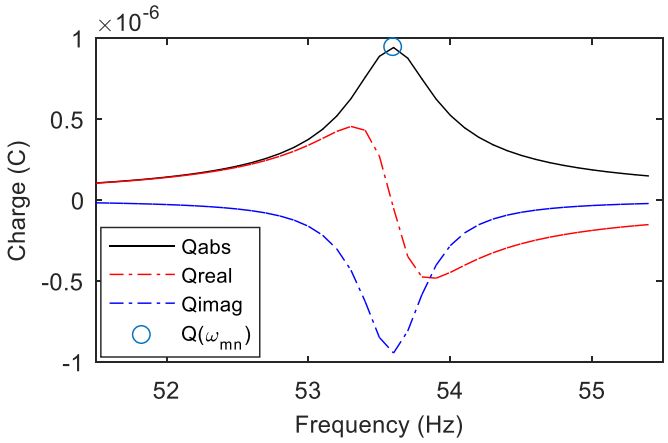


Figure 18. Charge response of the piezo patch at the excitation frequency for single patch harvester.

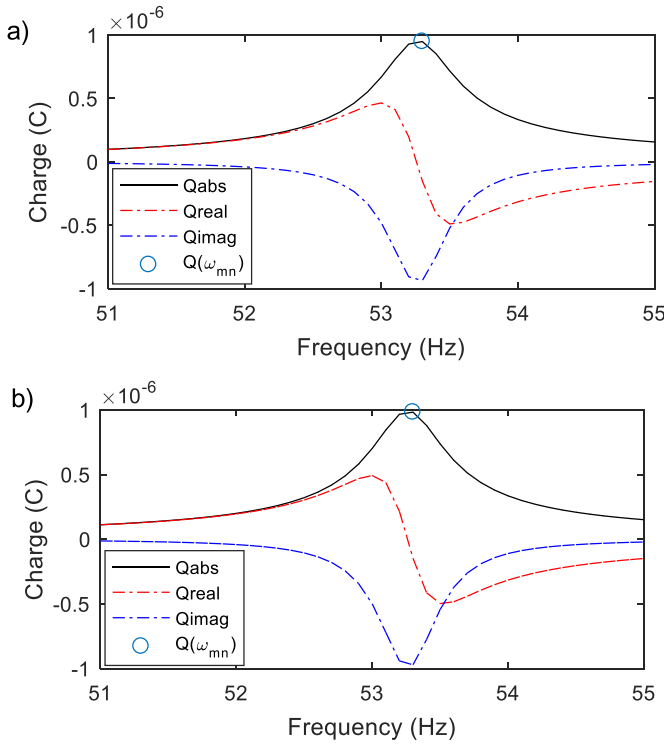


Figure 19. Charge response of piezo patches for RGU ECM identification method (a) PZT-1 and (b) PZT-2.

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